Two-Photon Fields: Coherence, Interference and Entanglement

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Introduction/Outline

One Photon Interference Introduction

- Introduction to coherence

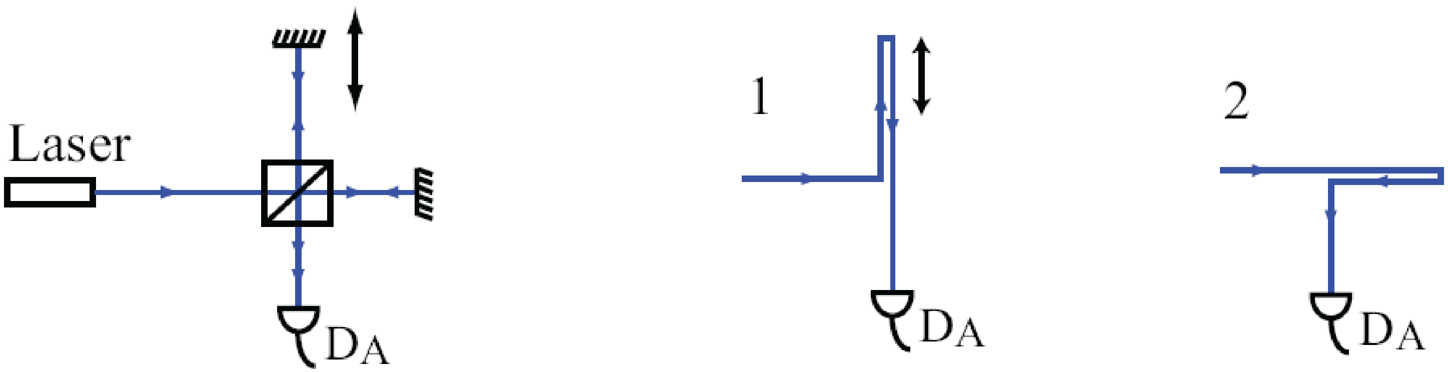
Parametric Down-conversion as a source of entangled photons

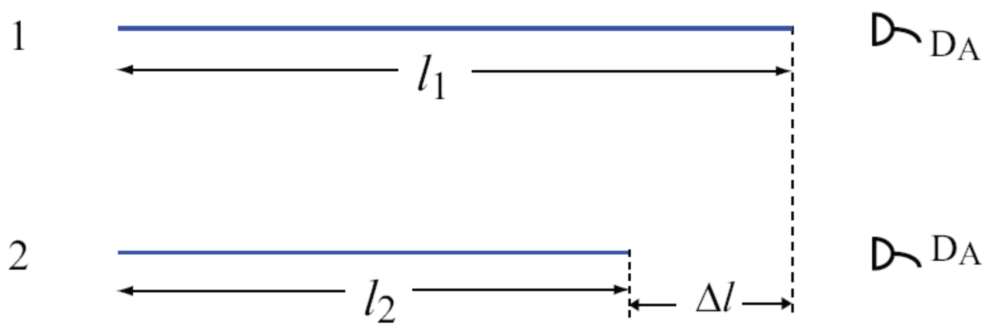
- Phase matching condition, Quantum Entanglement, Bell inequalities, Quantum Information

Two-Photon Interference

- Temporal, Spatial, and Angular.

One-Photon Interference: "A photon interferes with itself " - Dirac





$$\Delta l = l_1 - l_2$$

$$I_A \propto \langle V_A^*(t) V_A(t) \rangle_t$$

$$I_\mathrm{A} \;\propto\; 1 + \gamma(\Delta l)\cos(k_0 \Delta l)$$

$$\gamma(\Delta l) = \frac{\langle V_1^*(t) V_2(t - \Delta l/c)\rangle_t}{\sqrt{|V_1(t)|^2 |V_2(t)|^2}}$$

Necessary condition for interference:

$$\boxed{\Delta l < l_\mathrm{coh}}$$

Mandel and Wolf,
Optical Coherence and Quantum Optics

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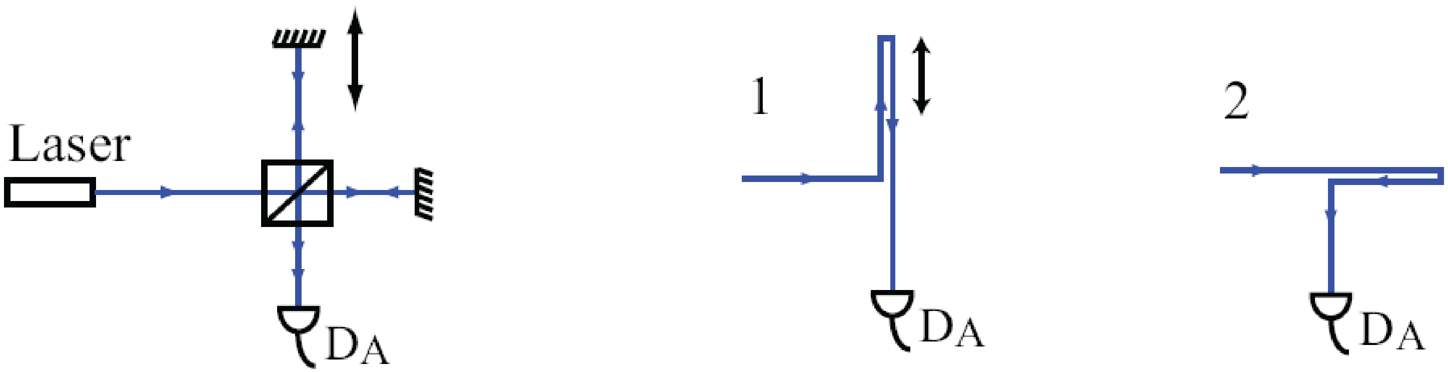

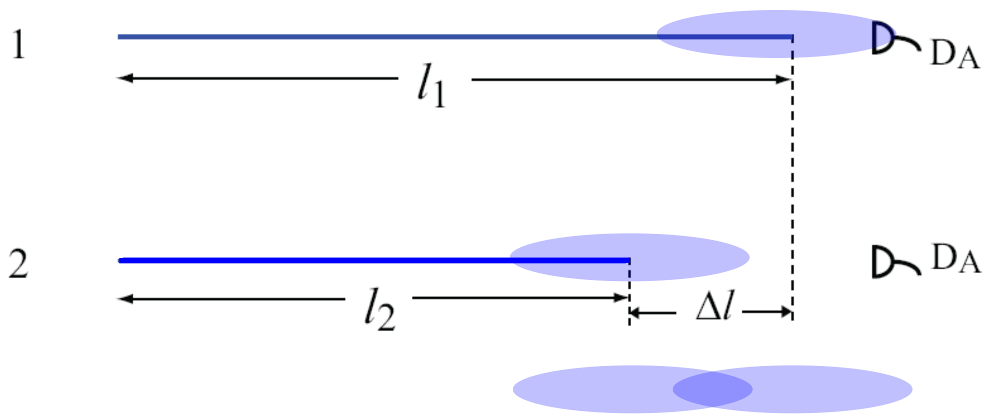

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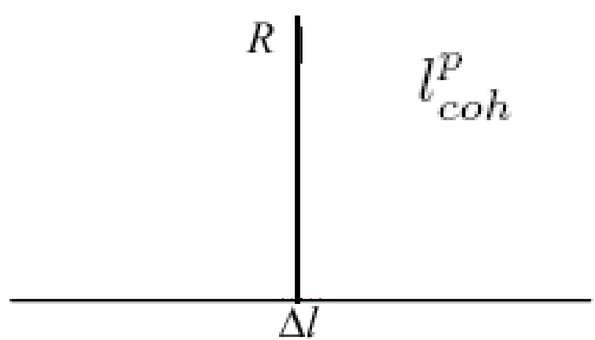

Necessary condition for interference:

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Mandel and Wolf,
Optical Coherence and Quantum Optics

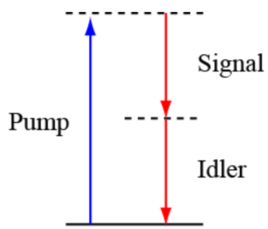
Parametric down-conversion

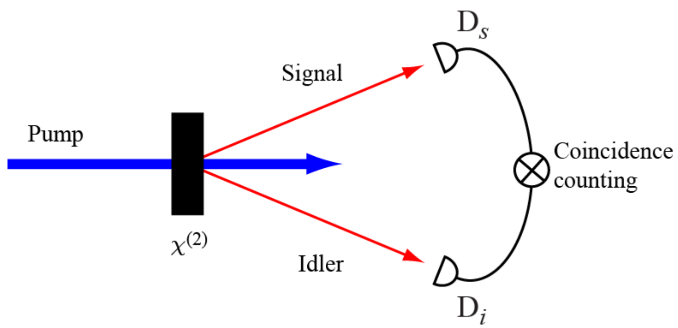
$$P(\boldsymbol{r},t)=\epsilon_0\chi^{(1)}E(\boldsymbol{r},t)$$

Leads to second-order nonlinear optical effects

$$P(\boldsymbol{r},t)=\epsilon_0\chi^{(1)}E(\boldsymbol{r},t)+\epsilon_0\chi^{(2)}E^2(\boldsymbol{r},t)+\epsilon_0\chi^{(3)}E^3(\boldsymbol{r},t)+\cdots$$

$$H(t)=\frac{1}{2}\int_{\mathcal{V}}d^3\boldsymbol{r}P^{(2)}(\boldsymbol{r},t)\cdot E(\boldsymbol{r},t)$$





$$\hat{H}(t')=\frac{\epsilon_0}{2}\int_{\mathcal{V}}d^3\boldsymbol{r}\chi^{(2)}\hat{E}_p^{(+)}(\boldsymbol{r},t')\hat{E}_s^{(-)}(\boldsymbol{r},t')\hat{E}_i^{(-)}(\boldsymbol{r},t')+\text{H.c.}$$

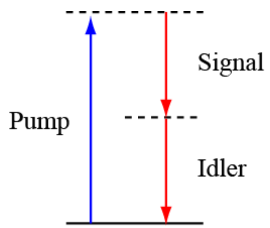
$$|\psi(0)\rangle=|\text{vac}\rangle_s|\text{vac}\rangle_i+\frac{1}{i\hbar}\int_{-t_{\text{int}}}^{0}dt'\hat{H}(t')|\text{vac}\rangle_s|\text{vac}\rangle_i$$

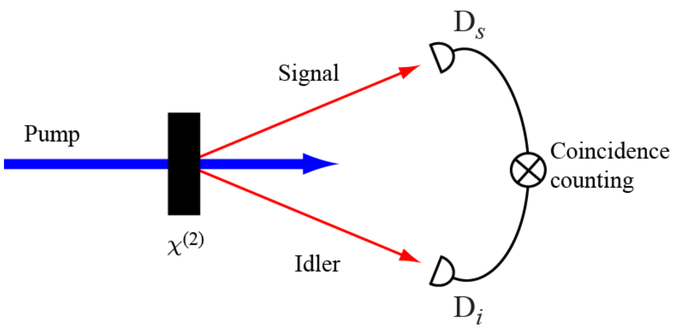
$$|\psi_{\text{tp}}\rangle\neq|\psi\rangle_s\otimes|\psi\rangle_i$$

two-photon field

$$|\psi_{\text{tp}}\rangle=\iint d\omega_p d\omega_s\phi(\omega_p,\omega_s)|\omega_s\rangle_s|\omega_p-\omega_s\rangle_i$$

Conservation laws and entanglement





$\omega_p = \omega_s + \omega_i$ **Entanglement in time and energy**

"Temporal" two-photon coherence

$\boldsymbol{q}_p = \boldsymbol{q}_s + \boldsymbol{q}_i$ **Entanglement in position and momentum**

"Spatial" two-photon coherence

$l_p = l_s + l_i$ **Entanglement in angular position and orbital angular momentum**

"Angular" two-photon coherence

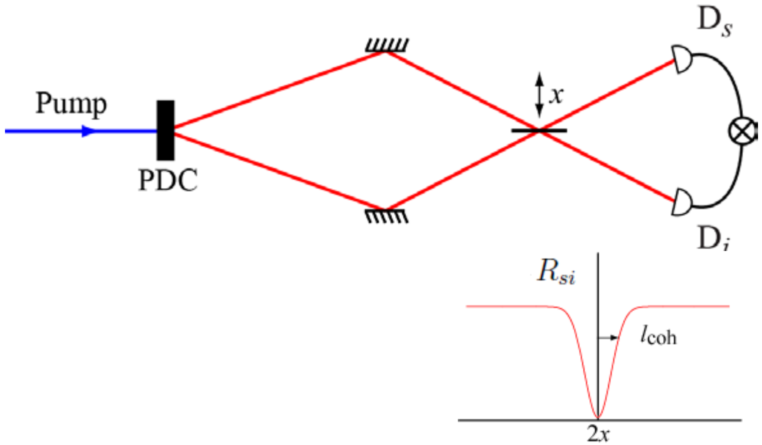
Coherence length of pump laser: l^p_{coh} ~ 10 cms.

Coherence length of signal-idler field: l_{coh} ~ $c/\Delta\omega$ ~ 100 μm.

Two-Photon Interference

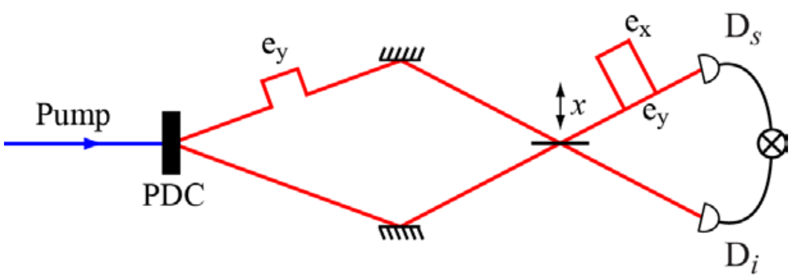
- **Hong-Ou-Mandel effect**

C. K. Hong et al., PRL 59, 2044 (1987)



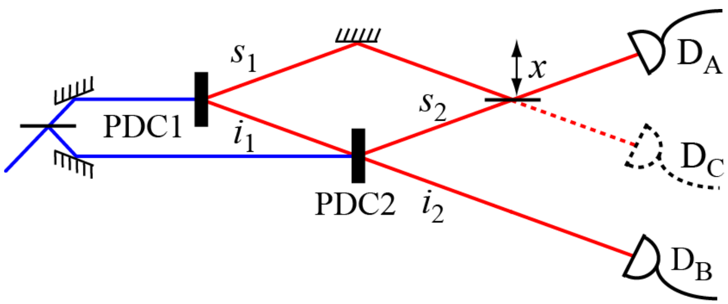
- **Postponed Compensation Experiment**

T. B. Pittman, PRL 77, 1917 (1996)



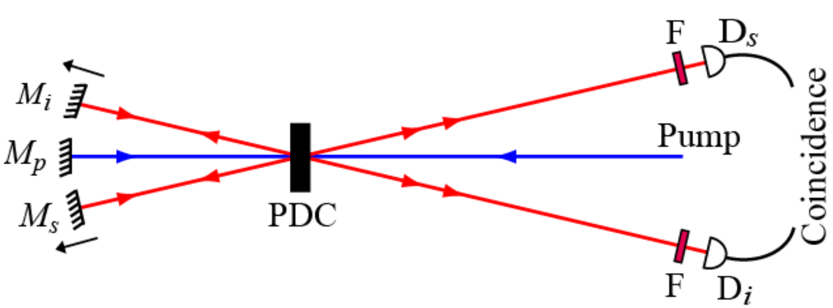
- **Induced Coherence**

X. Y. Zou et al., PRL 67, 318 (1991)

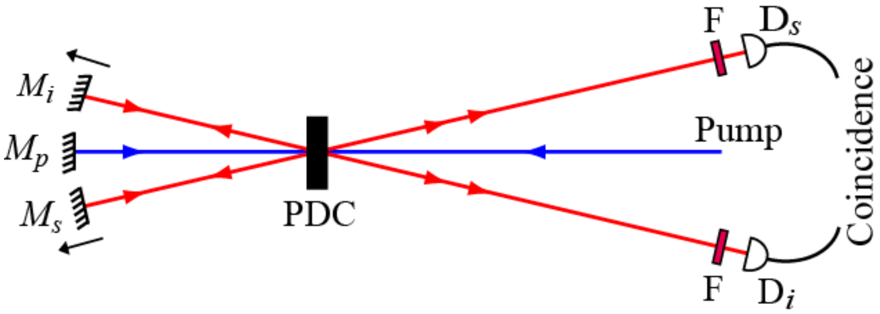


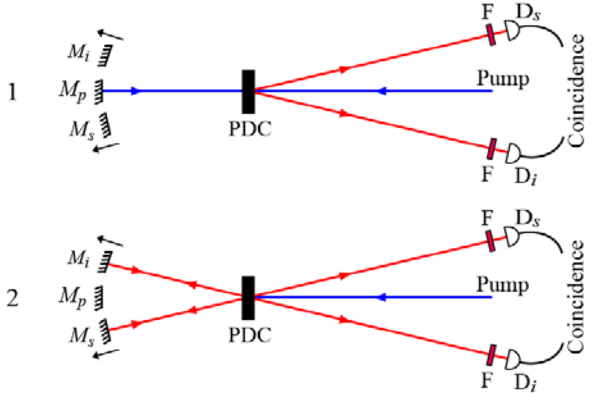
- **Frustrated two-photon Creation**

T. J. Herzog et al., PRL 72, 629 (1994)



Two-Photon Interference: A two-photon interferes with itself





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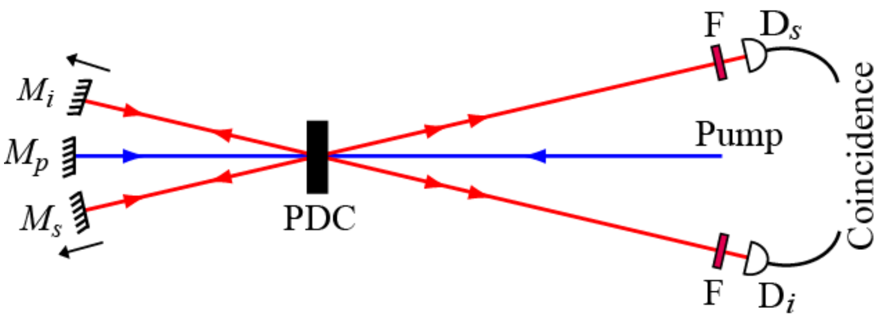

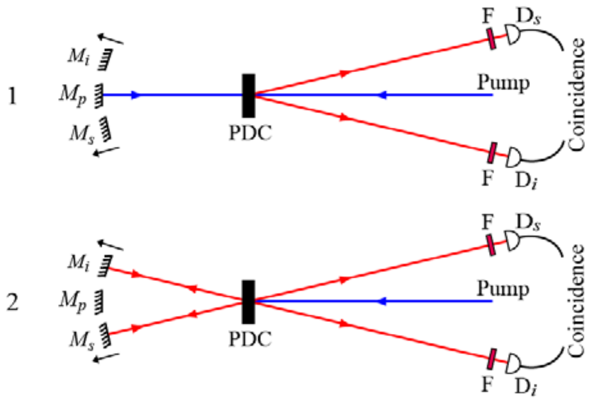

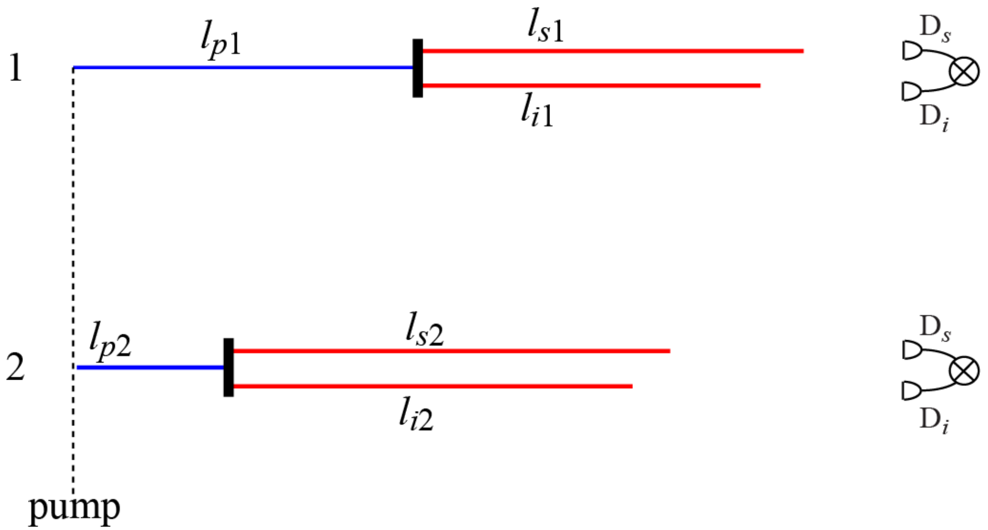

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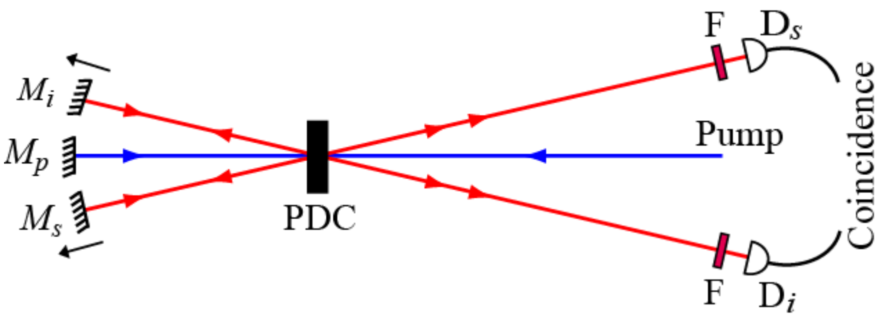

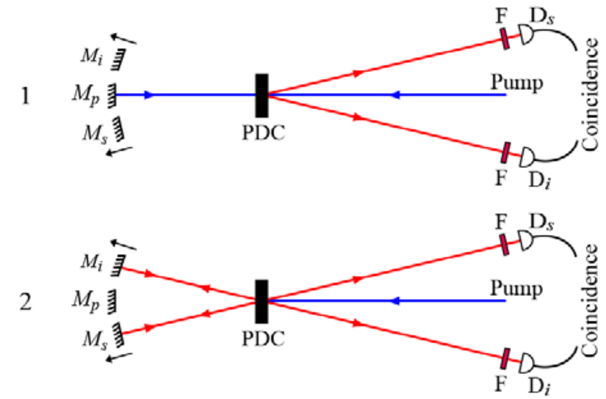

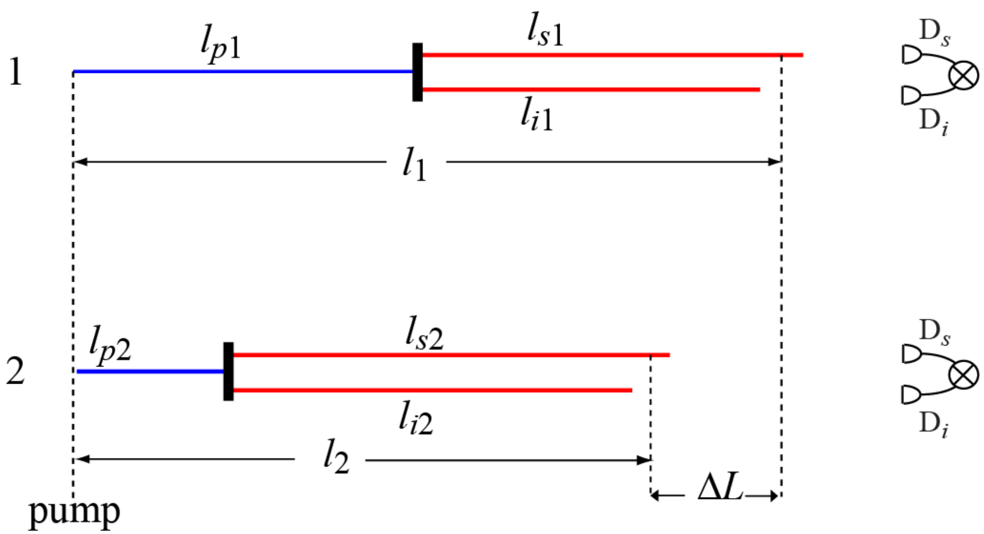

$$\Delta L \equiv l_1 - l_2$$

two-photon path-length difference

Jha, O'Sullivan, Chan, and Boyd et al., PRA **77**, 021801(R) (2008)

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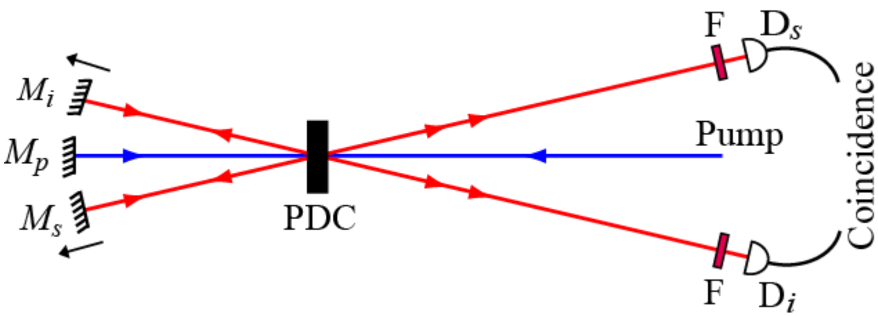

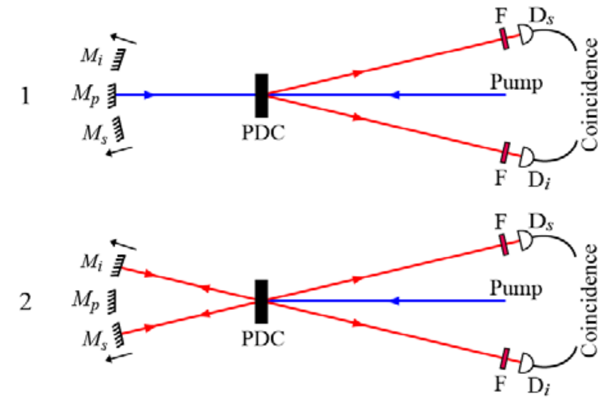

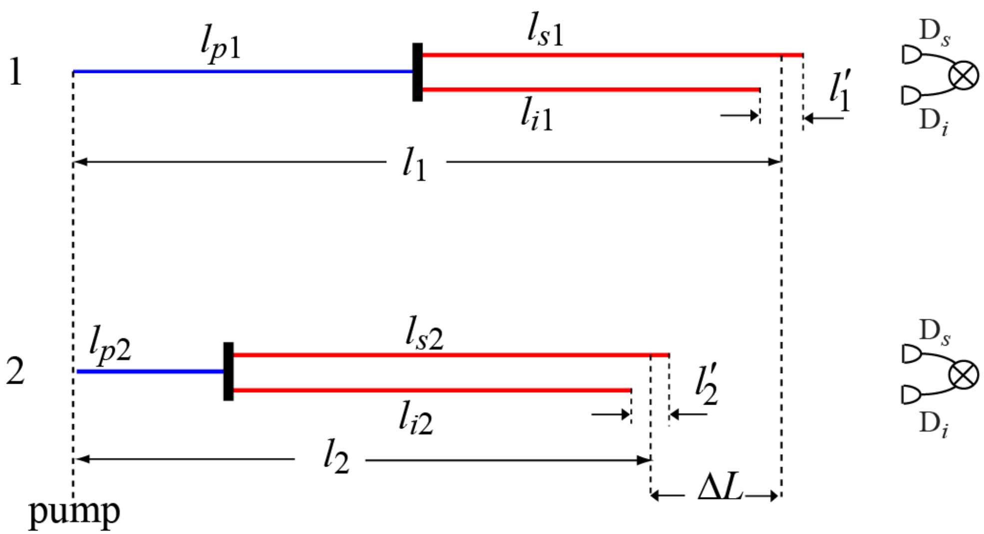

$$\Delta L \equiv l_1 - l_2$$

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$$\Delta L' \equiv l'_1 - l'_2$$

two-photon path-asymmetry length difference

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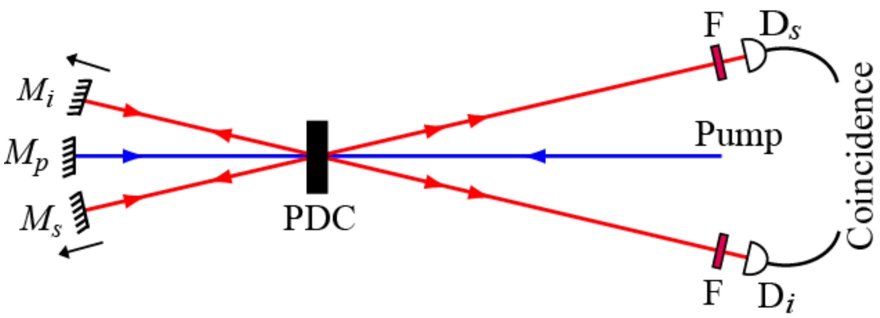

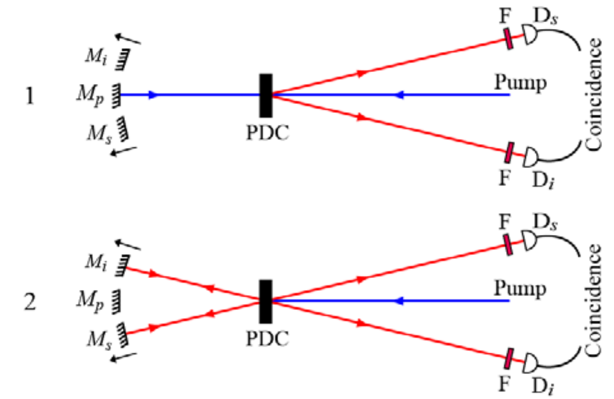

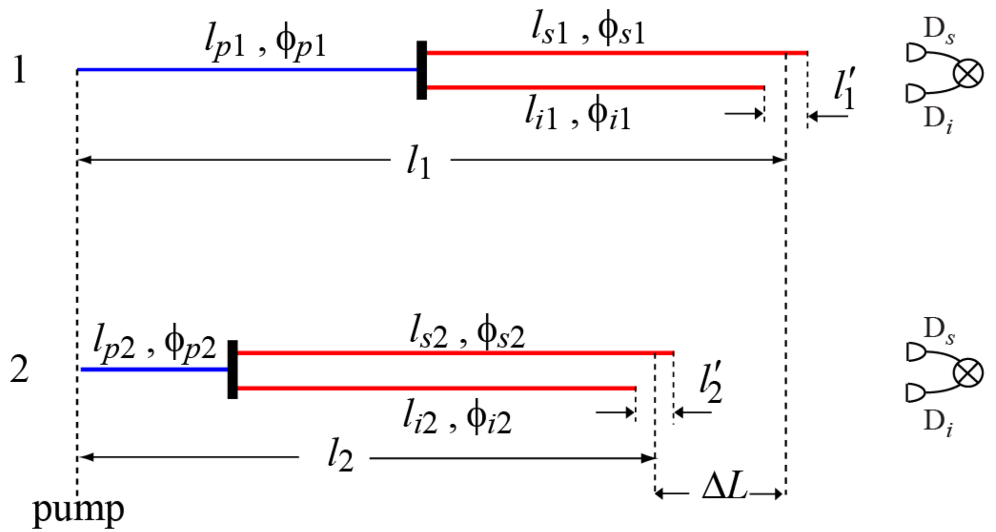

$$\Delta L \equiv l_1 - l_2$$

two-photon path-length difference

$$\Delta L' \equiv l_1' - l_2'$$

two-photon path-asymmetry length difference

$$\Delta\phi \equiv (\phi_{s1} + \phi_{i1} + \phi_{p1}) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$

Jha, O'Sullivan, Chan, and Boyd et al., PRA **77**, 021801(R) (2008)

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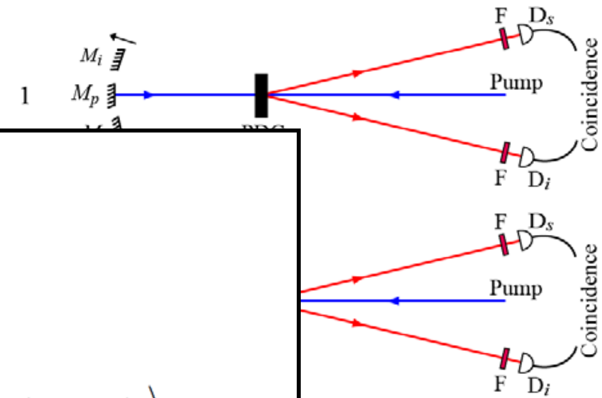

R. J. Glauber, Phys. Rev. **130**, 2529 (1963)

$$R_{si} = \left\langle \langle\psi|\hat{E}_s^{(-)}(t)\hat{E}_i^{(-)}(t+\tau)\hat{E}_i^{(+)}(t+\tau)\hat{E}_s^{(+)}(t)|\psi\rangle \right\rangle_{t,\tau}$$

$$|\psi\rangle = A\iint_0^\infty d\omega_p d\omega_s V_1(\omega_p)\Phi_1(\omega_s,\omega_p-\omega_s)e^{i(\omega_p\tau_{p1}+\phi_{p1})}|\omega_s\rangle_{s1}|\omega_p-\omega_s\rangle_{i1}$$
$$+A\iint_0^\infty d\omega_p d\omega_s V_2(\omega_p)\Phi_2(\omega_s,\omega_p-\omega_s)e^{i(\omega_p\tau_{p2}+\phi_{p2})}|\omega_s\rangle_{s2}|\omega_p-\omega_s\rangle_{i2}$$

$$\hat{E}_s^{(+)}(t) = \int_0^\infty d\omega f_s(\omega-\omega_{s0})$$
$$\times\left[c_{s1}\hat{a}_{s1}(\omega)e^{-i[\omega(t-\tau_{s1})-\phi_{s1}]} + c_{s2}\hat{a}_{s2}(\omega)e^{-i[\omega(t-\tau_{s2})-\phi_{s2}]}\right]$$

$$\hat{E}_i^{(+)}(t) = \int_0^\infty d\omega' f_i(\omega'-\omega_{i0})$$
$$\times\left[c_{i1}\hat{a}_{i1}(\omega')e^{-i[\omega'(t-\tau_{i1})-\phi_{i1}]} + c_{i2}\hat{a}_{i2}(\omega')e^{-i[\omega'(t-\tau_{i2})-\phi_{i2}]}\right]$$

difference

metry length difference

$\phi_{p1}) - (\phi_{s2}+\phi_{i2}+\phi_{p2})$

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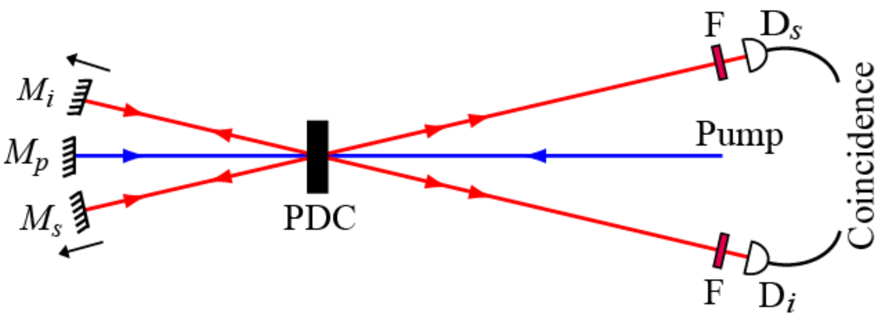

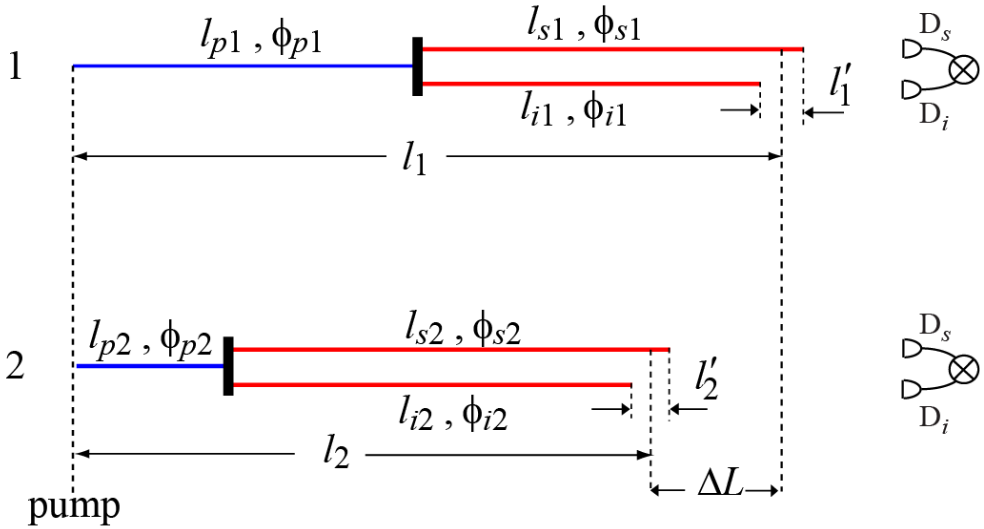

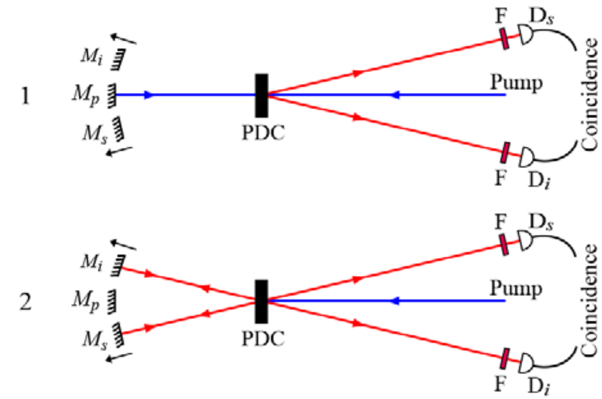

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two-photon path-length difference

$$\Delta L' \equiv l'_1 - l'_2$$

two-photon path-asymmetry length difference

$$\Delta\phi \equiv (\phi_{s1} + \phi_{i1} + \phi_{p1}) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$

$$R_{si} = C[1 + \gamma'(\Delta L')\,\gamma(\Delta L)\cos(k_0\Delta L + \Delta\phi)]$$

$$\gamma(\Delta L) = \frac{\langle v_1(t) v_2^*(t + \Delta L/c)\rangle_t}{\sqrt{|v_1|^2|v_2|^2}} \qquad \gamma'(\Delta L') = \frac{\langle g_1^*(\tau) g_2(\tau - \Delta L'/c)\rangle_\tau}{\sqrt{|g_1|^2|g_2|^2}}$$

Necessary conditions for two-photon interference:

$$\boxed{\begin{aligned} \Delta L &< l_{\text{coh}}^p \\ \Delta L' &< l_{\text{coh}} \end{aligned}}$$

Jha, O'Sullivan, Chan, and Boyd et al., PRA **77**, 021801(R) (2008)

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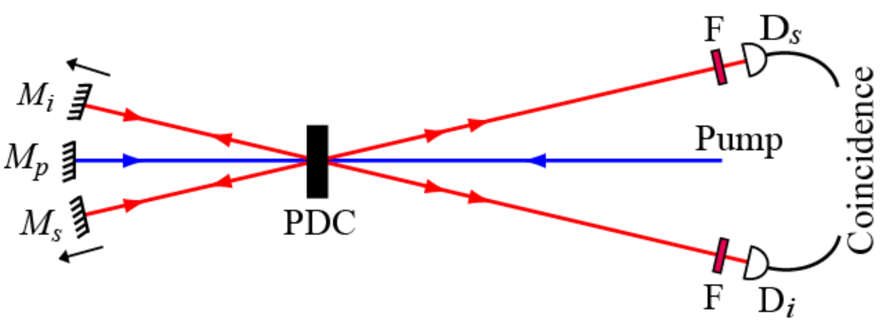

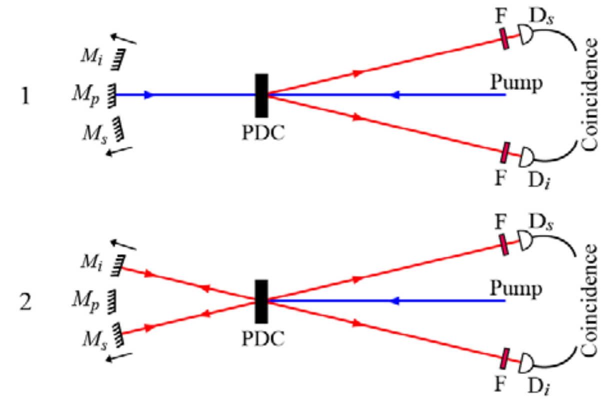

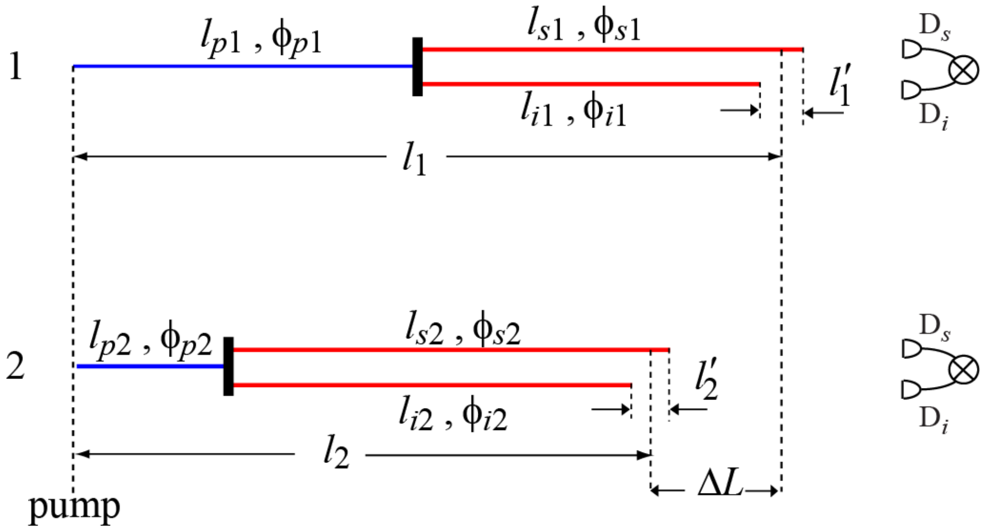

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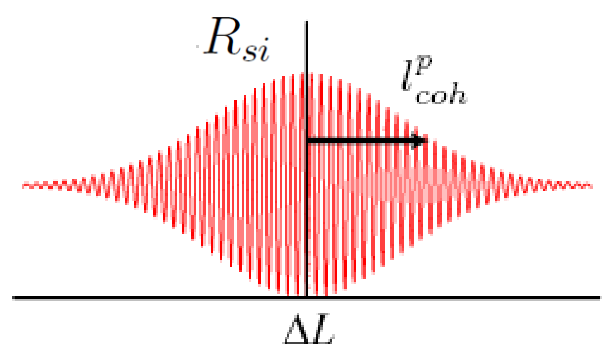
$$R_{si} = C[1 + \gamma'(\Delta L')\,\gamma(\Delta L)\cos(k_0 \Delta L + \Delta\phi)]$$

Case I : $\Delta L' = 0$

- **ΔL plays the same role in two-photon interference as Δl does in one-photon interference**



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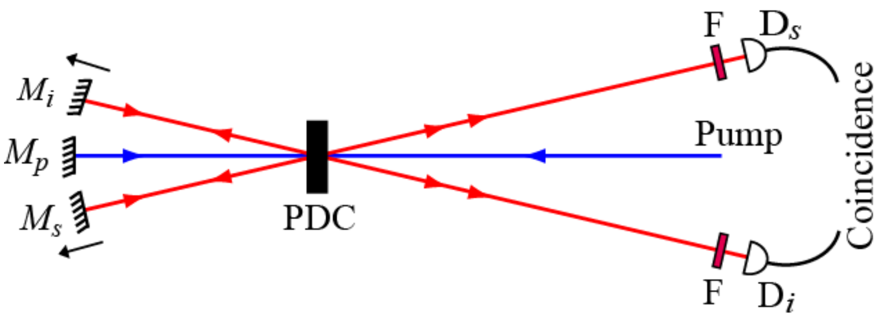

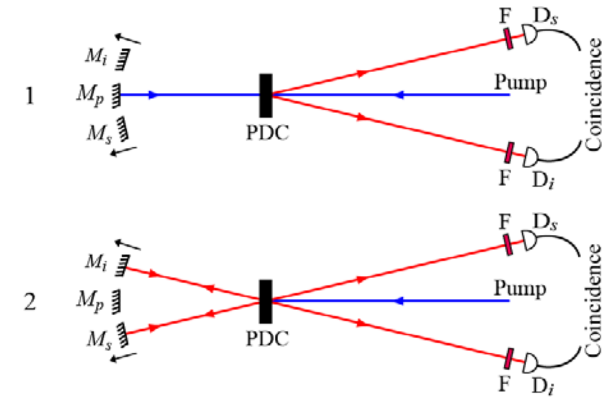

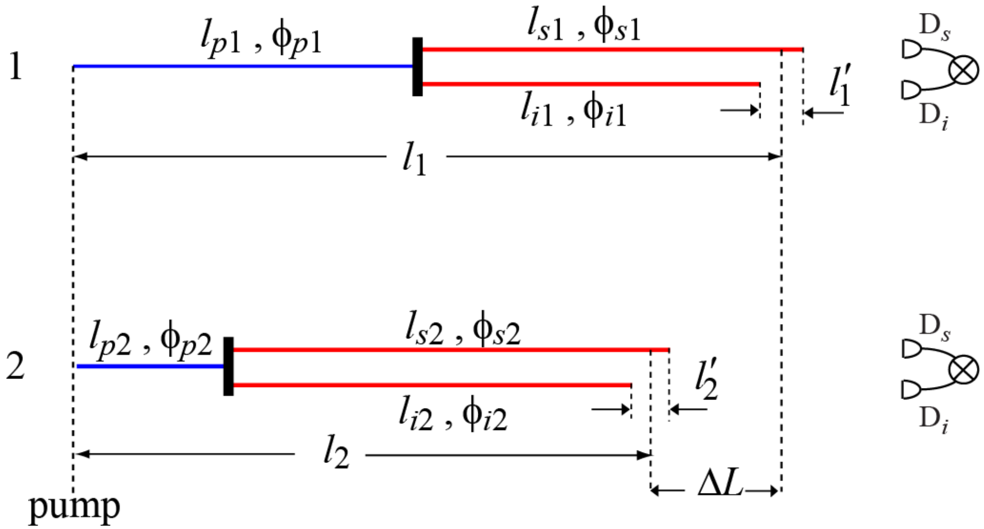

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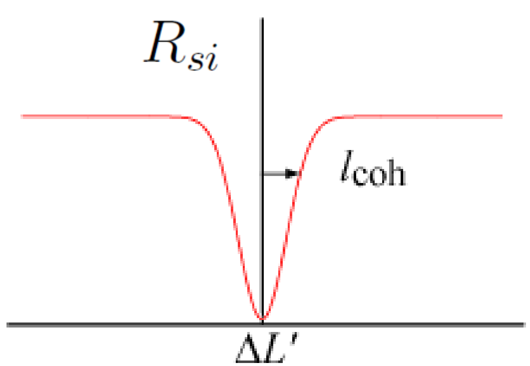
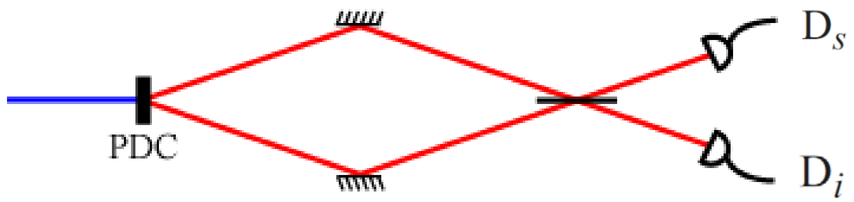
$$R_{si} = C[1 + \gamma'(\Delta L')\,\gamma(\Delta L)\cos(k_0 \Delta L + \Delta\phi)]$$

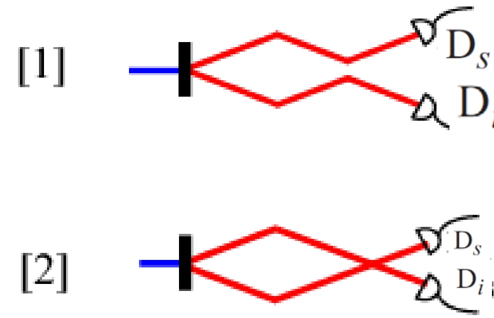
Case I : ΔL = 0

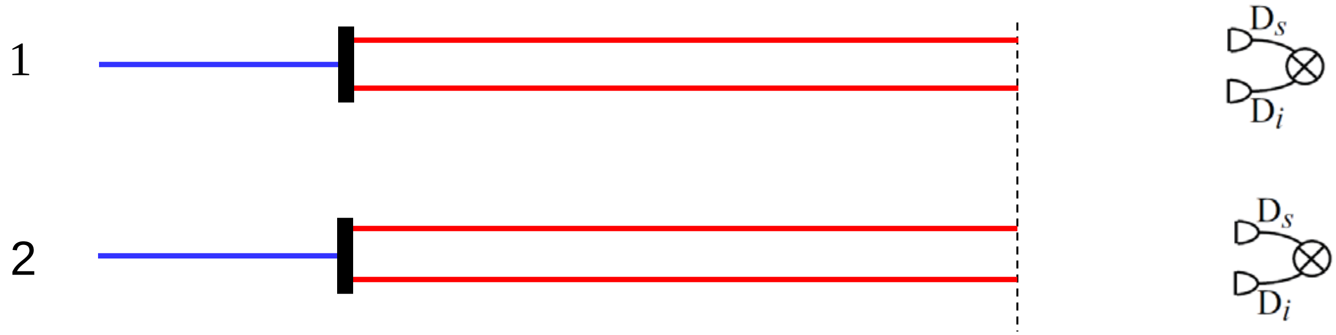
- **$\Delta L'$ has no one-photon analog**
- **The curve represents how coherence is lost due to an increase in the two-photon path-length asymmetry difference $\Delta L'$**



Hong-Ou-Mandel (HOM) Effect







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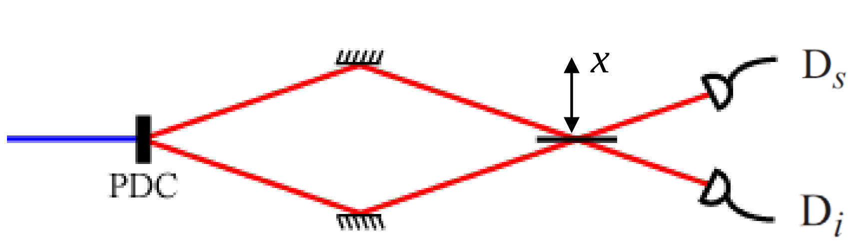
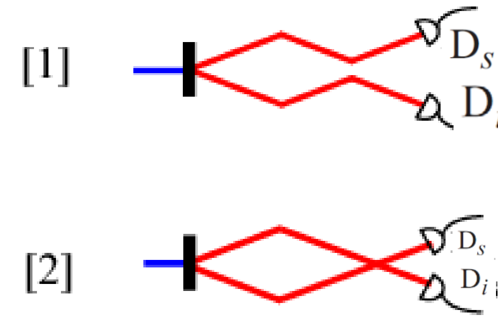
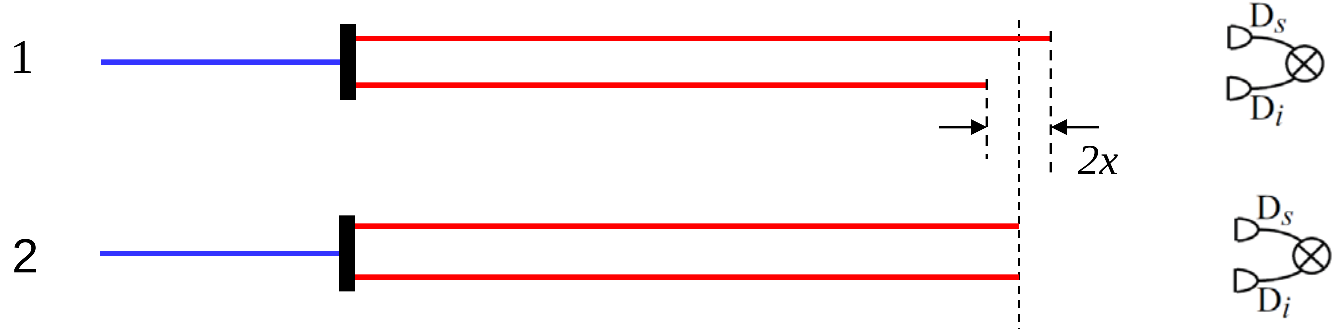
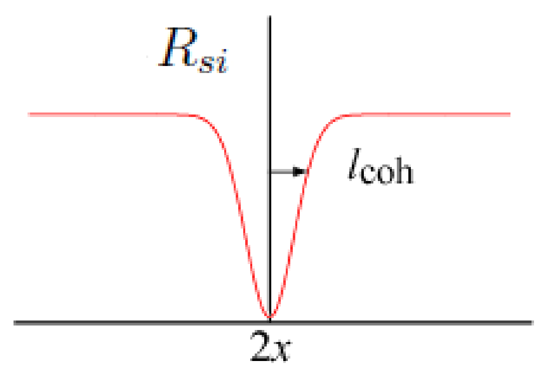




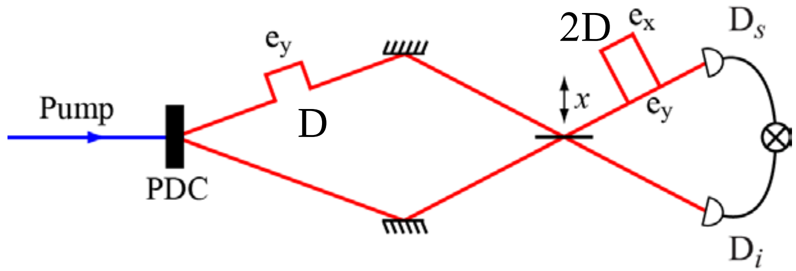


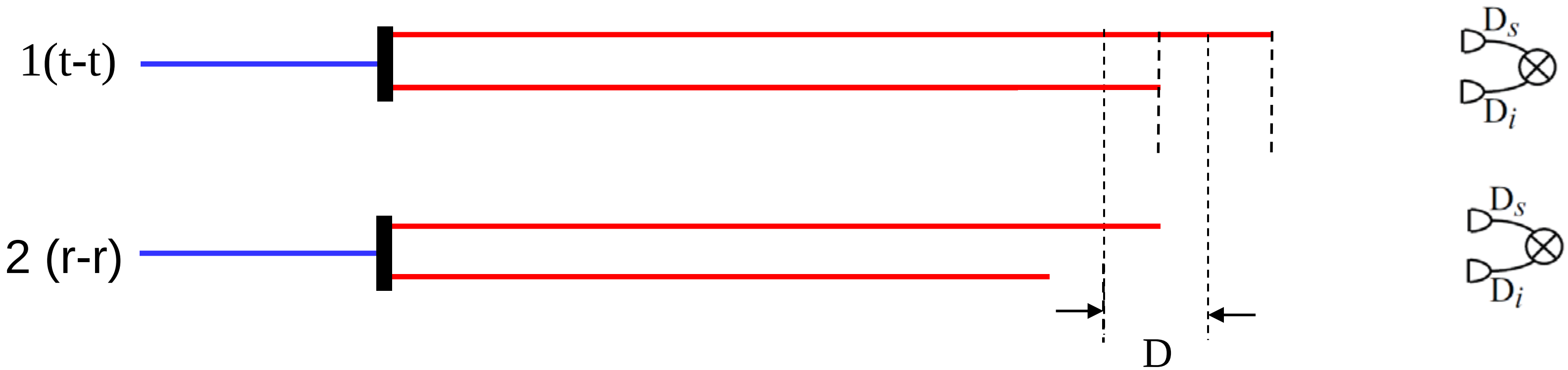
$\Delta L = 0$; $\Delta L' = 2x$

$$R_{si} = C[1 - \gamma'(2x)]$$



Postponed Compensation Experiment





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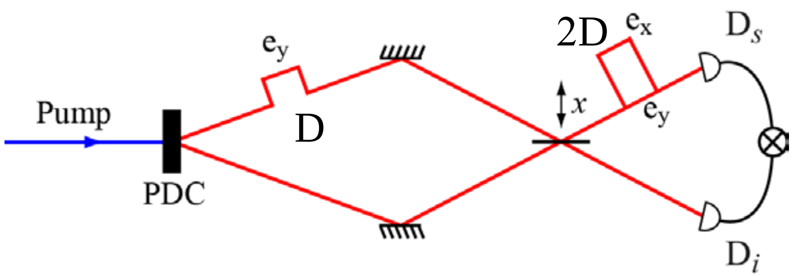
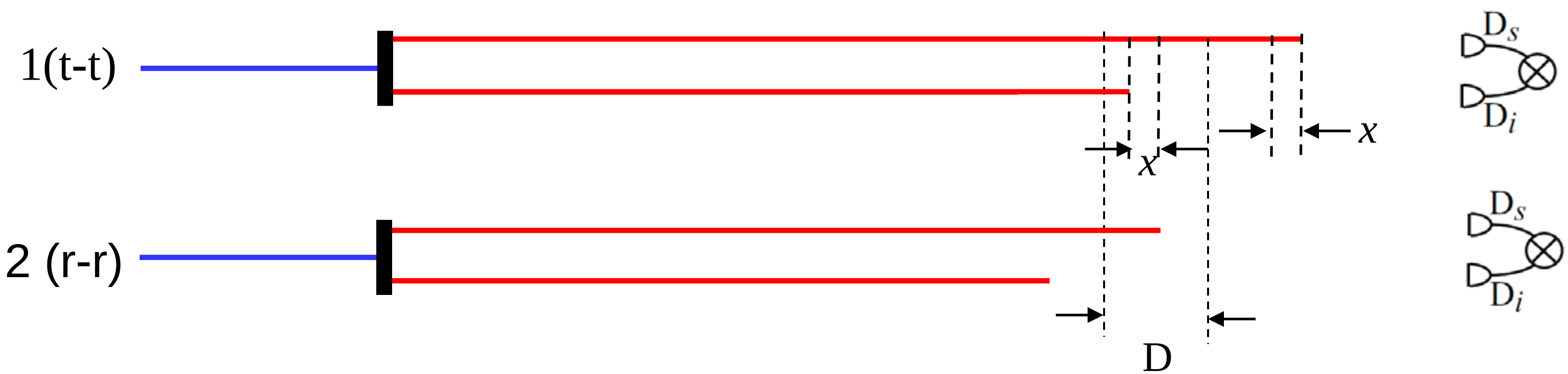
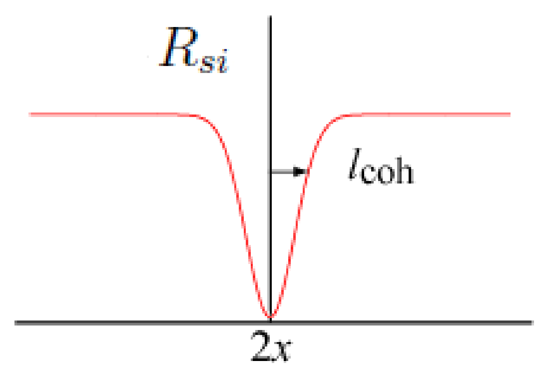


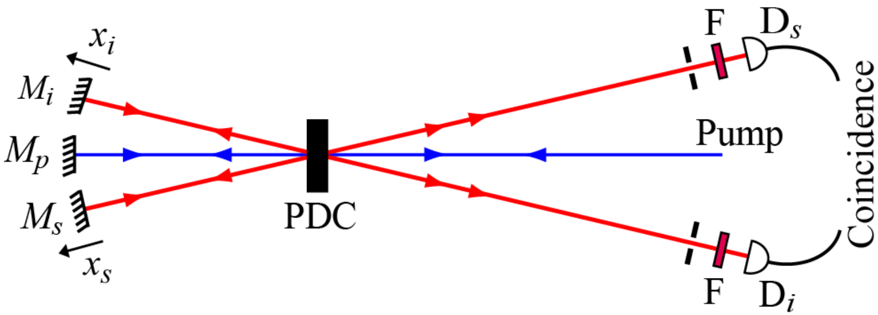


$$\Delta L = D\,; \quad \Delta L' = 2x$$

$$R_{si} = C[1 - \gamma'(2x)\,\gamma(D)\cos(k_0 D)]$$

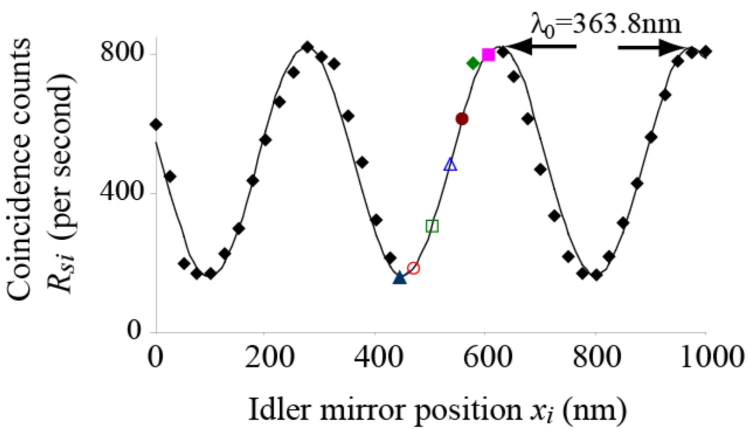


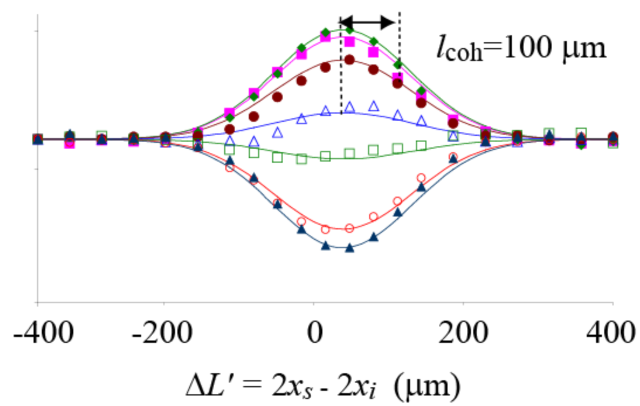
Experimental Verification



$$\Delta L = x_s + x_i \qquad \Delta L' = 2x_s - 2x_i \qquad \gamma(\Delta L) \sim 1$$

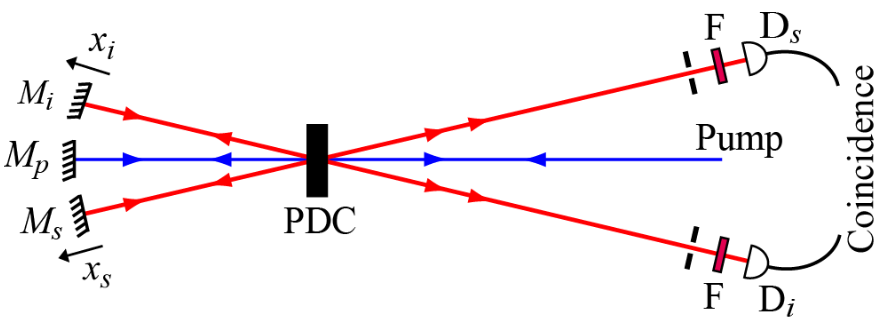
$$R_{si} = C\{\ 1 - \gamma'(2x_s - 2x_i)\cos\left[k_0(x_s + x_i)\right]\}$$





Jha, O'Sullivan, Chan, and Boyd et al., PRA **77**, 021801(R) (2008)

Experimental Verification

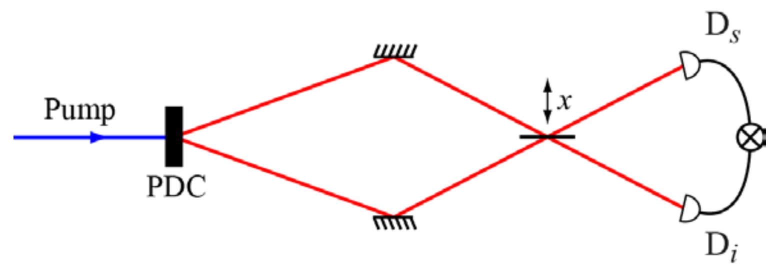


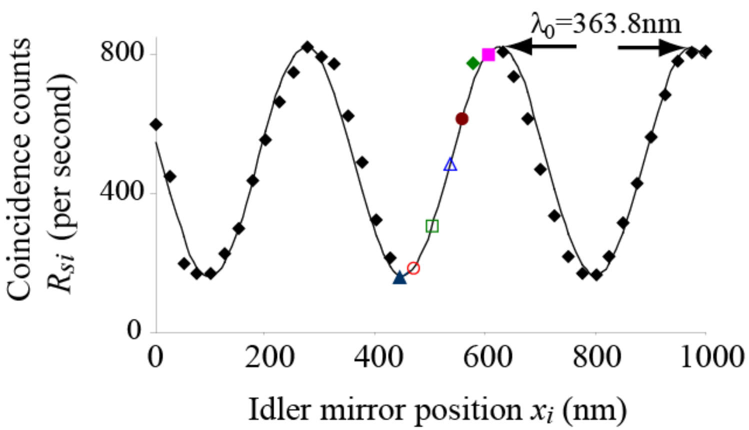
$\Delta L = x_s + x_i \qquad \Delta L' = 2x_s - 2x_i \qquad \gamma(\Delta L) \sim 1$

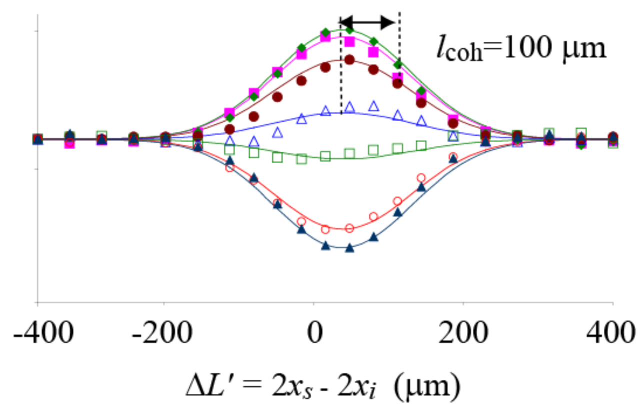
$$R_{si} = C\{\ 1 - \gamma'(2x_s - 2x_i)\cos\left[k_0(x_s + x_i)\right]\}$$

- **Hong-Ou-Mandel effect**

C. K. Hong et al., PRL 59, 2044 (1987)

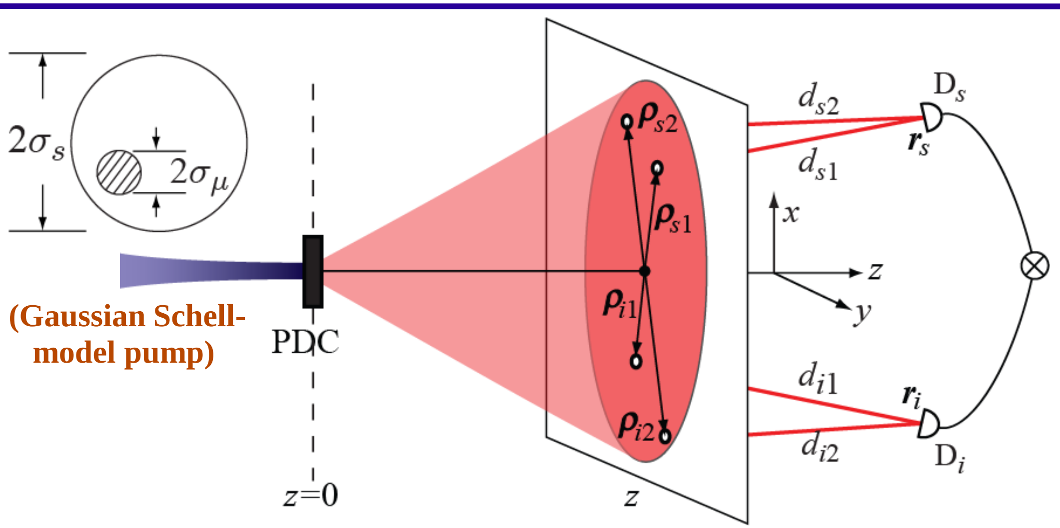


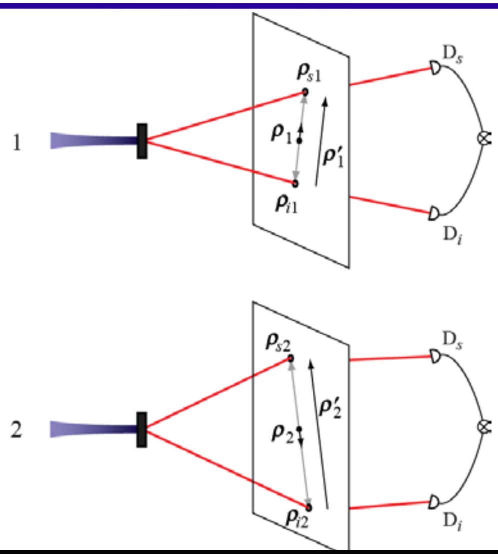




Jha, O'Sullivan, Chan, and Boyd et al., PRA **77**, 021801(R) (2008)

Spatial Two-photon Interference





Two-photon cross-spectral density:

$$W^{(2)}(\boldsymbol{\rho}_{s1}, \boldsymbol{\rho}_{i1}, \boldsymbol{\rho}_{s2}, \boldsymbol{\rho}_{i2}, z) = \mathrm{tr}\{\rho_{\mathrm{tp}} \hat{E}_{s1}^{(-)}(\boldsymbol{r}_{s1}) \hat{E}_{i1}^{(-)}(\boldsymbol{r}_{i1}) \hat{E}_{i2}^{(+)}(\boldsymbol{r}_{i2}) \hat{E}_{s2}^{(+)}(\boldsymbol{r}_{s2})\}$$

Two-photon transverse position vector :

$$\boldsymbol{\rho}_1 \equiv \frac{\boldsymbol{\rho}_{s1} + \boldsymbol{\rho}_{i1}}{2}, \quad \boldsymbol{\rho}_2 \equiv \frac{\boldsymbol{\rho}_{s2} + \boldsymbol{\rho}_{i2}}{2}; \quad \Delta\boldsymbol{\rho} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$$

Two-photon position-asymmetry vector :

$$\boldsymbol{\rho}_1' \equiv \boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{i1}, \quad \boldsymbol{\rho}_2' \equiv \boldsymbol{\rho}_{s2} - \boldsymbol{\rho}_{i2}; \quad \Delta\boldsymbol{\rho}' = \boldsymbol{\rho}_1' - \boldsymbol{\rho}_2'$$

$$|W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)| = \sqrt{S^{(2)}(\boldsymbol{\rho}_1, z) S^{(2)}(\boldsymbol{\rho}_2, z)} \mu^{(2)}(\Delta\boldsymbol{\rho}, z)$$

$$S^{(2)}(\boldsymbol{\rho}_1, z) = C \exp\left\{-(1/2)\left[\boldsymbol{\rho}_1/\sigma_s^{(2)}(z)\right]^2\right\} \qquad \sigma_s^{(2)}(z) = \sigma_s(z)$$

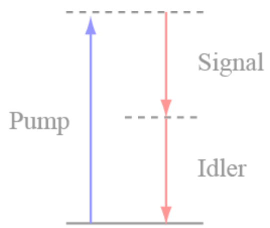
$$\mu^{(2)}(\Delta\boldsymbol{\rho}, z) = \exp\left\{-(1/2)\left[\Delta\boldsymbol{\rho}/\sigma_\mu^{(2)}(z)\right]^2\right\} \qquad \sigma_\mu^{(2)}(z) = \sigma_\mu(z)$$

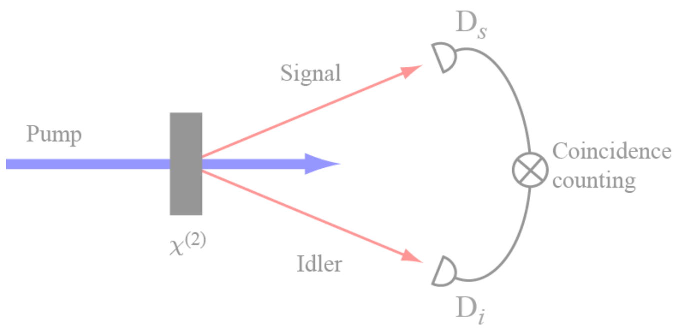
Necessary condition for interference:

$$\boxed{|\Delta\boldsymbol{\rho}| < \sigma_\mu(z)}$$

A. K. Jha and R.W. Boyd, PRA **81,** 013828 (2010)

Conservation laws and entanglement





$$\omega_p = \omega_s + \omega_i$$

Entanglement in time and energy

"Temporal" two-photon coherence

$$q_p = q_s + q_i$$

Entanglement in position and momentum

"Spatial" two-photon coherence

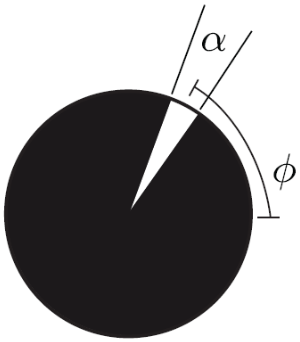
$$l_p = l_s + l_i$$

Entanglement in angular position and orbital angular momentum

"Angular" two-photon coherence

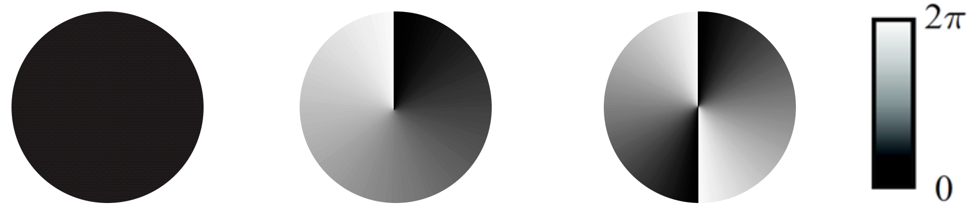
Angular Fourier Relationship

Angular position

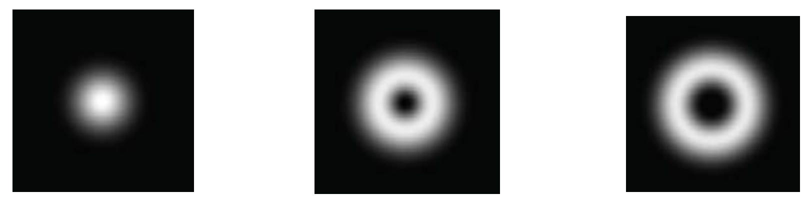


Laguerre-Gauss basis LG_p^l

$$\mathbf{A} = \hat{x} u(\rho, z) e^{-ikz} e^{il\phi}$$



***l*=0** ***l*=1** ***l*=2**



$$A_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi(\phi) \exp(-il\phi)$$

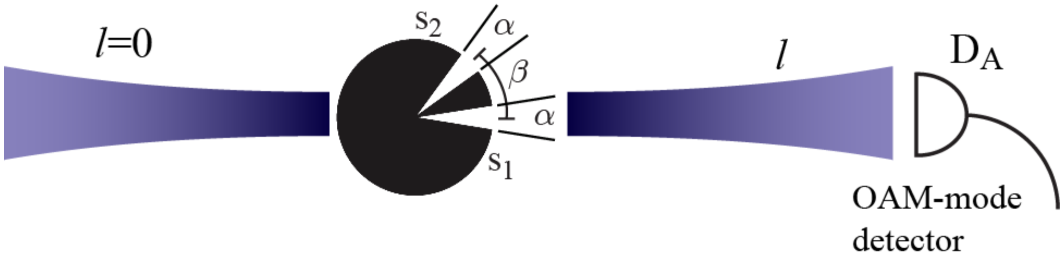
$$\Psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{+\infty} A_l \exp(il\phi)$$

Barnett and Pegg, PRA **41**, 3427 (1990)
Franke-Arnold et al., New J. Phys. **6**, 103 (2004)
Forbes, Alonso, and Siegman J. Phys. A **36**, 707 (2003)

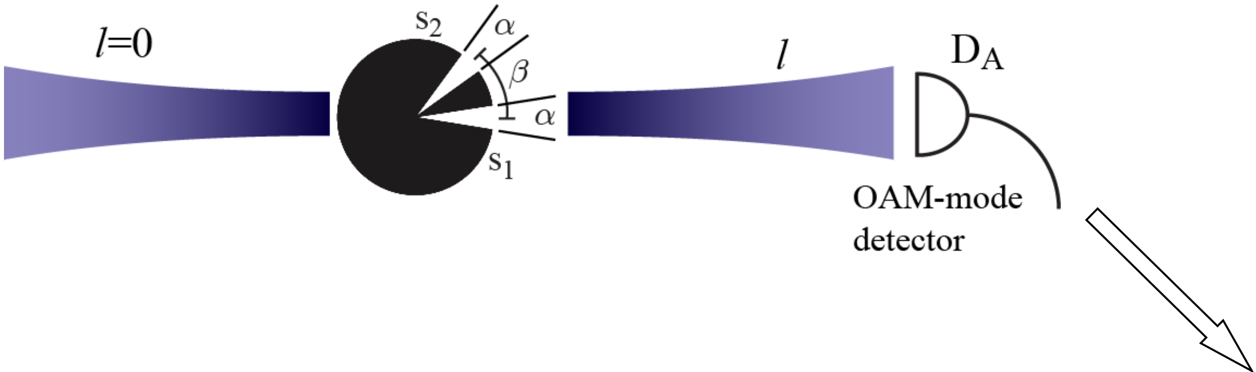
$$\frac{J_z}{W} = \frac{\iint \rho d\rho d\phi (\boldsymbol{\rho} \times \langle \mathbf{E} \times \mathbf{B} \rangle)_z}{c \iint \rho d\rho d\phi \langle \mathbf{E} \times \mathbf{B} \rangle_z} = \frac{\hbar l}{\hbar \omega}$$

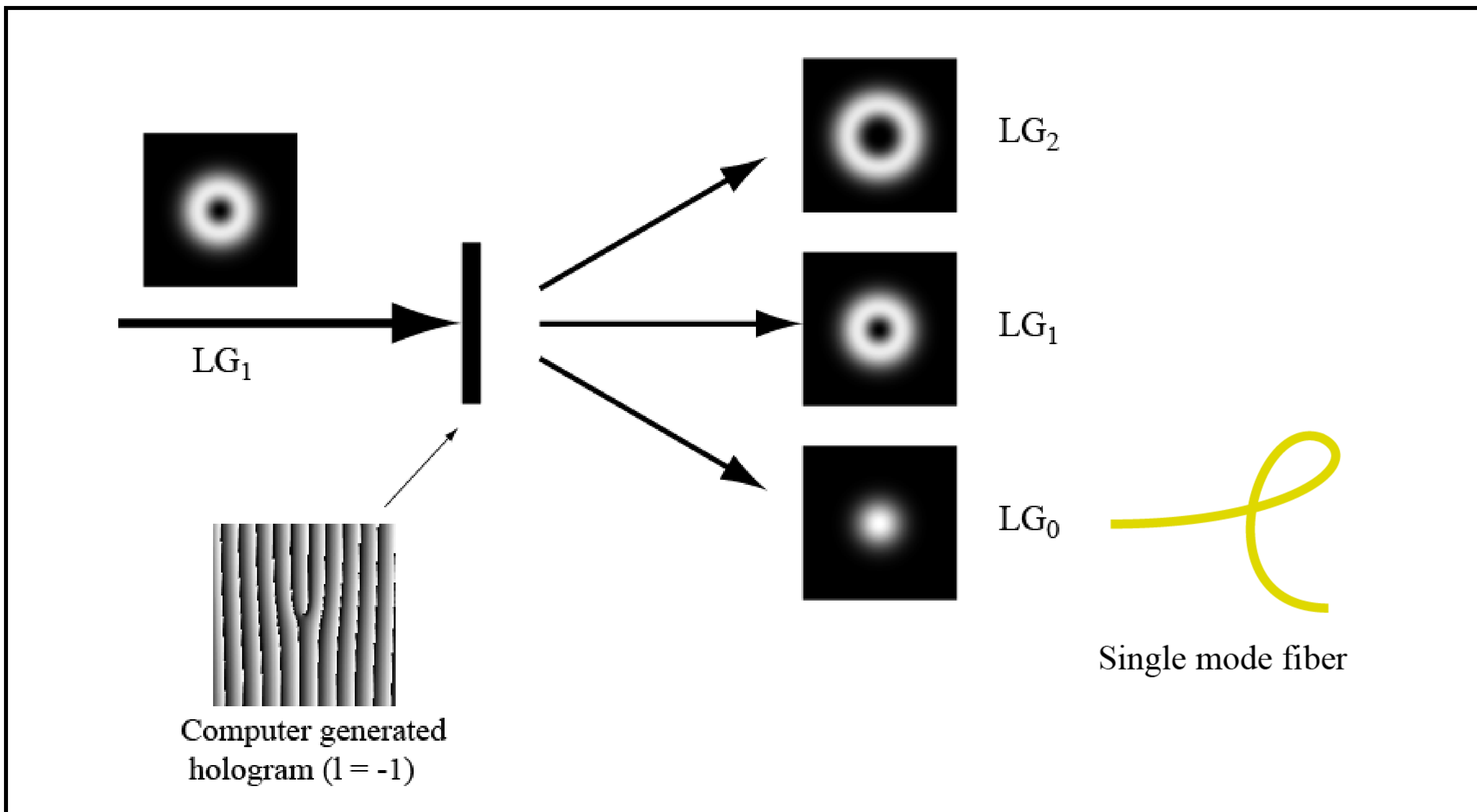
Allen et al., PRA **45**, 8185 (1992)

Angular One-Photon Interference

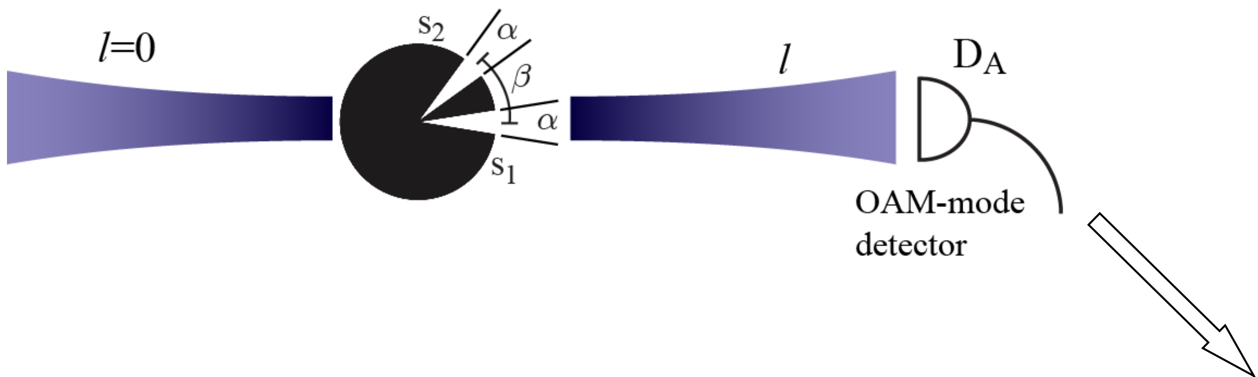


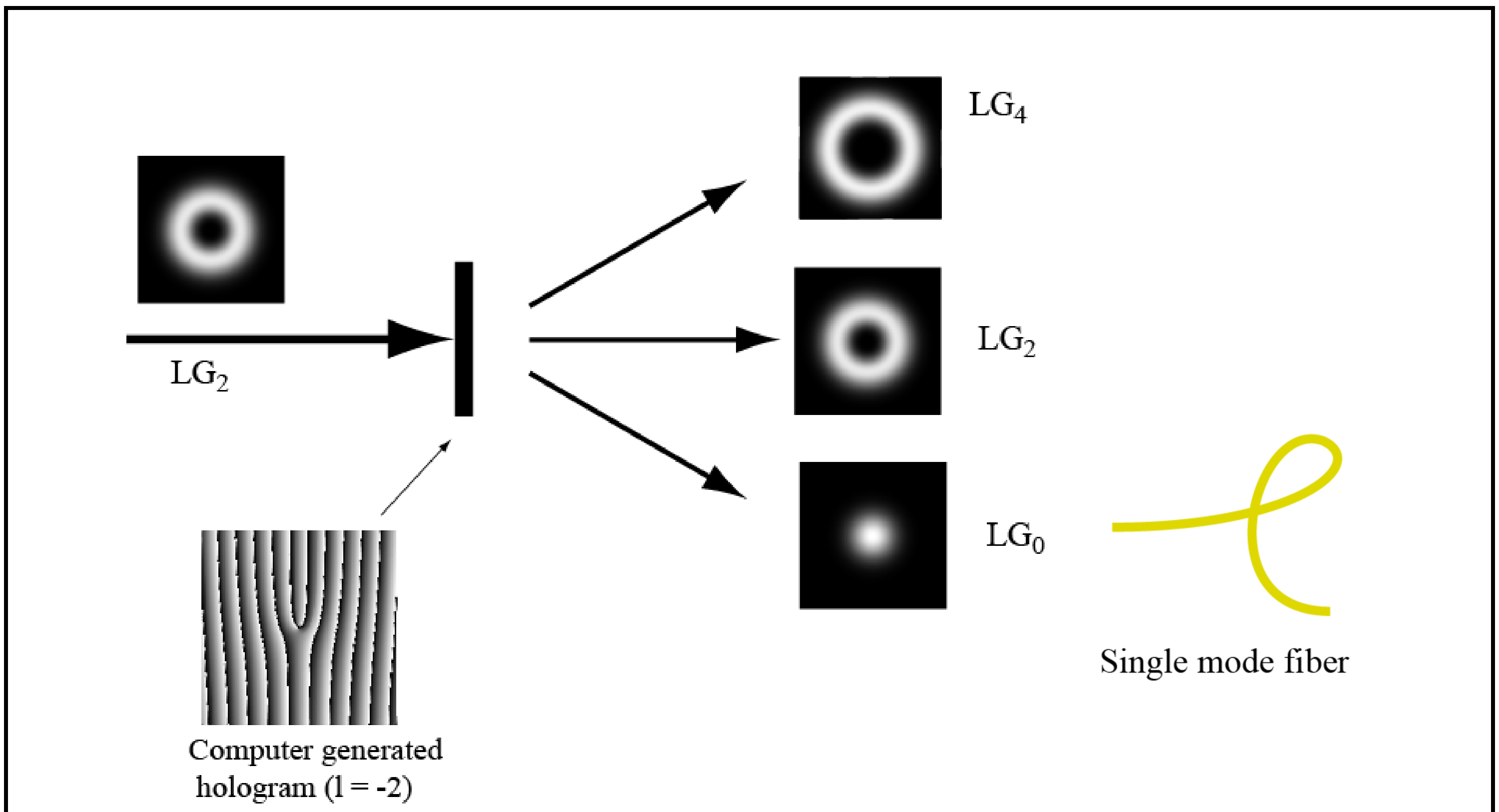
Angular One-Photon Interference



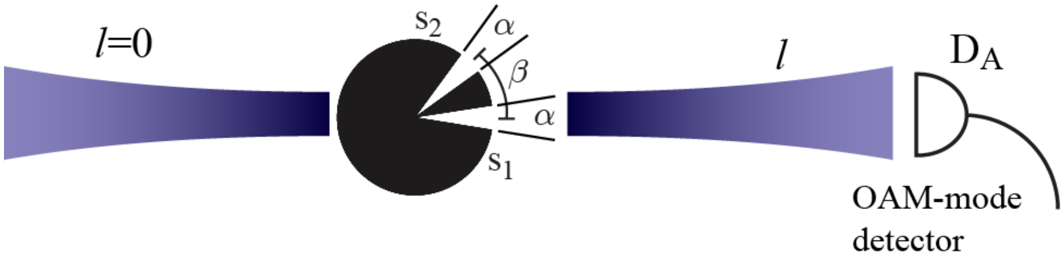


Angular One-Photon Interference





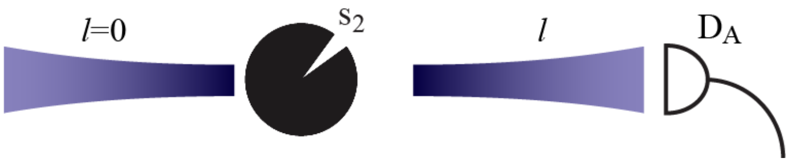
Angular One-Photon Interference



1

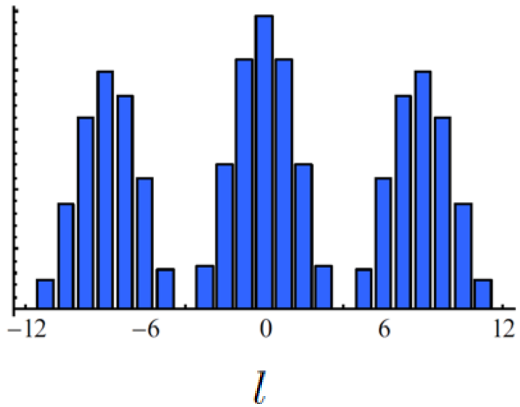
$$\psi_{1l} = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi_1(\phi) e^{-il\phi}$$
$$= \frac{\alpha}{\sqrt{2\pi}} \text{sinc}\left(\frac{l\alpha}{2}\right)$$

2



$$\psi_{2l} = \frac{\alpha}{\sqrt{2\pi}} \text{sinc}\left(\frac{l\alpha}{2}\right) e^{-il\beta}$$

$\alpha = \pi/10$
$\beta = \pi/4$

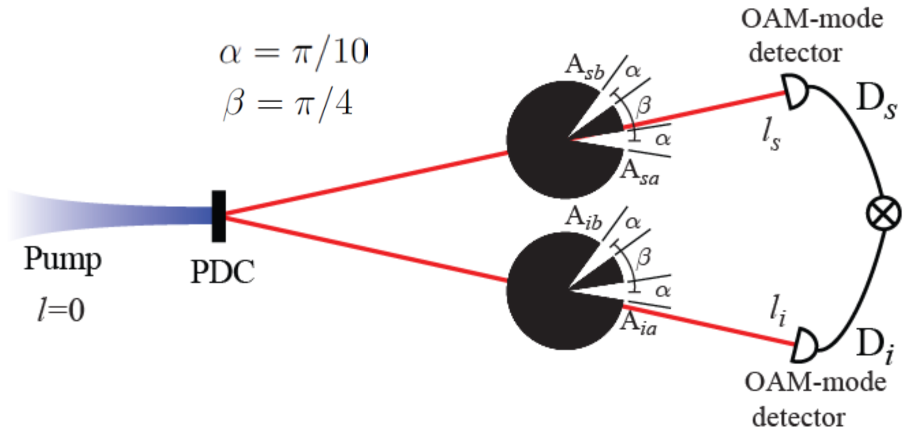


OAM-mode distribution:

$$I_A = C\frac{\alpha^2}{\pi} \text{sinc}^2\left(\frac{l\alpha}{2}\right) [1 + \cos(l\beta)]$$

E. Yao et al., Opt. Express **14**, 13089 (2006)
A. K. Jha, et al., PRA **78**, 043810 (2008)

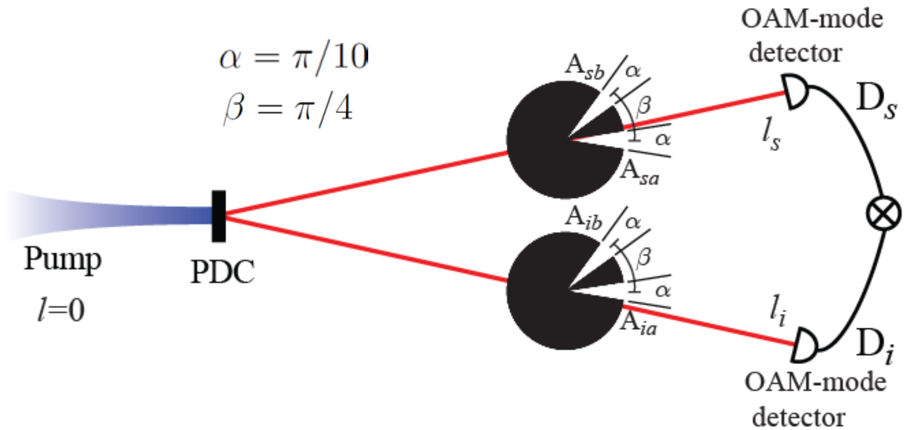
Angular Two-Photon Interference

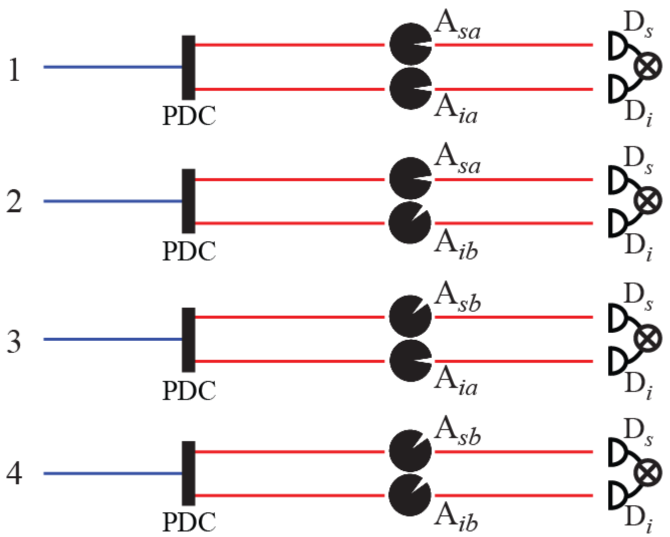


State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

Angular Two-Photon Interference





State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

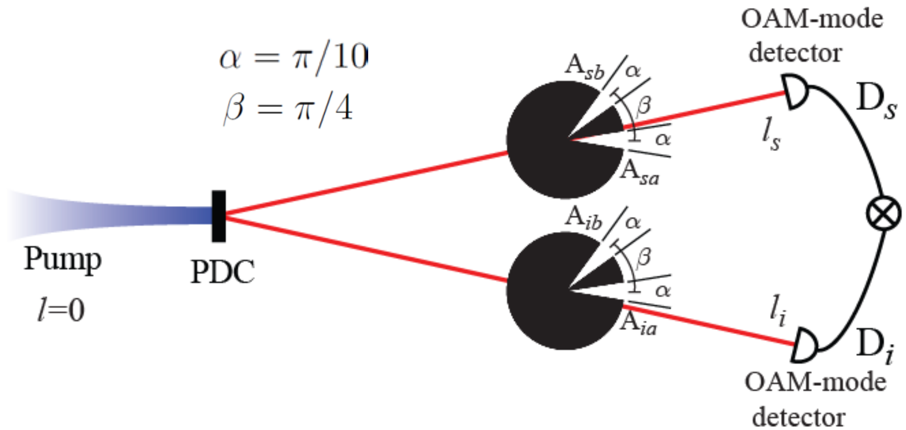
State of the two photons after the aperture:

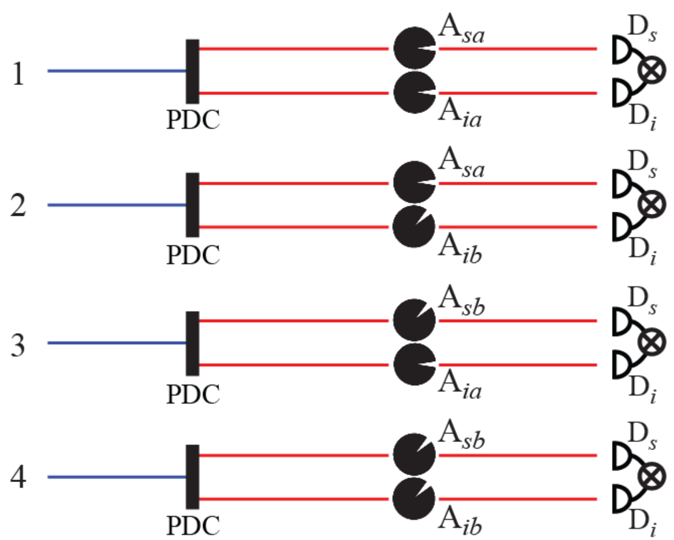
$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho_{41}^* = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

$$\rho_{11} + \rho_{44} = 1$$

Angular Two-Photon Interference





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Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right]\mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2 \times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\ \mu\cos[(l_s + l_i)\beta + \theta]\}$$

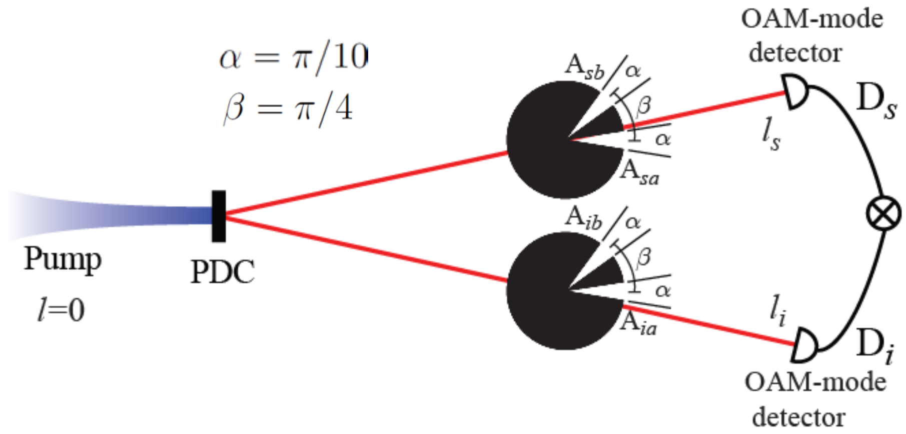
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

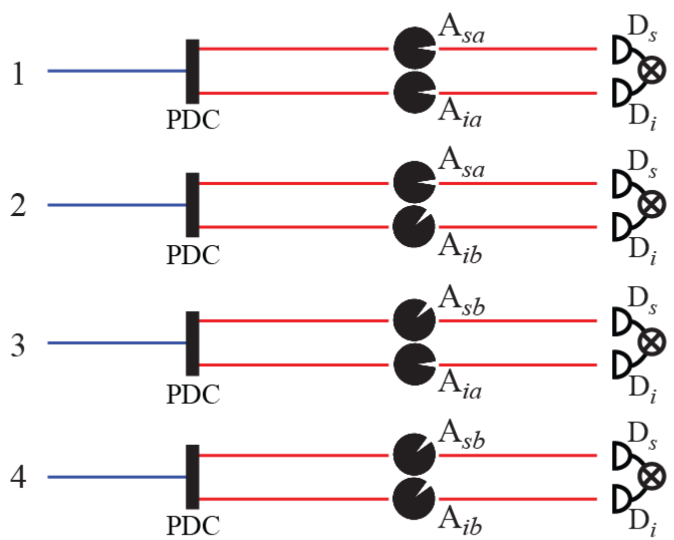
Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

$$\zeta = \rho_{\rm qubit}(\sigma_y \otimes \sigma_y)\rho_{\rm qubit}^*(\sigma_y \otimes \sigma_y)$$

$$C(\rho_{\rm qubit}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

Angular Two-Photon Interference





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$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix} \qquad \begin{aligned} \rho_{14} &= \rho_{41}^* \\ &= \sqrt{\rho_{11}\rho_{44}}\,\mu e^{i\theta} \\ \rho_{11} &+ \rho_{44} = 1 \end{aligned}$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right]\mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2$$
$$\times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\,\mu\cos[(l_s + l_i)\beta + \theta]\}$$

Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\,\mu$

Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

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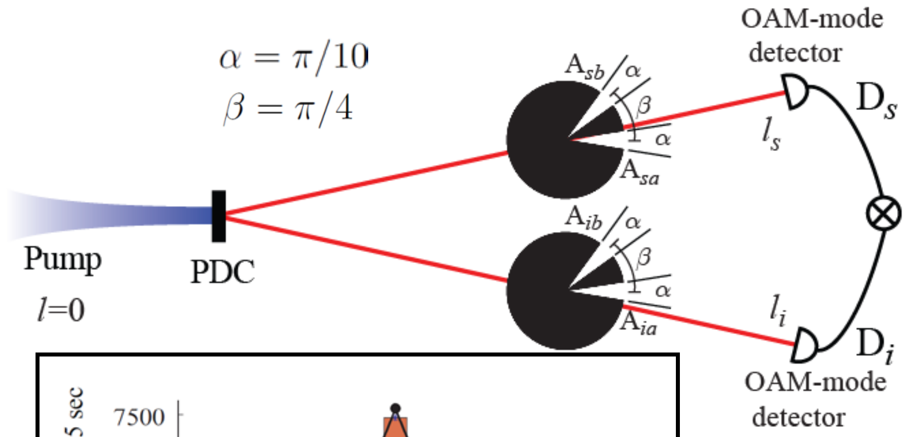
$$C(\rho_{\rm qubit}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

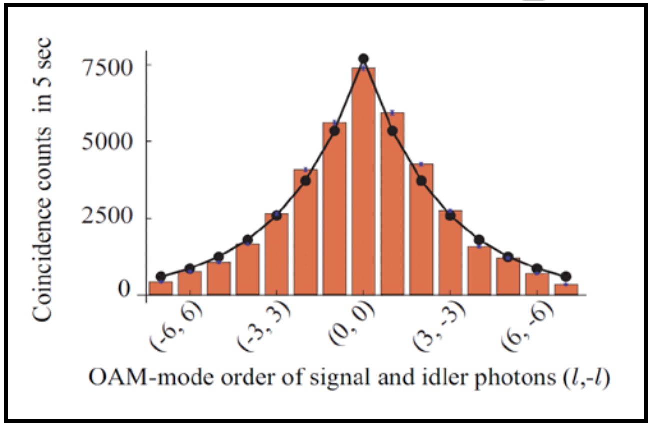
Concurrence of the two-qubit state:

$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\,\mu = V$$

A. K. Jha et al., PRL **104,** 010501 (2010)

Angular Two-Photon Interference





State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix} \qquad \begin{aligned} \rho_{14} &= \rho_{41}^* \\ &= \sqrt{\rho_{11}\rho_{44}}\,\mu e^{i\theta} \end{aligned}$$

$$\rho_{11} + \rho_{44} = 1$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,{\rm sinc}\left[(l_s - l)\frac{\alpha}{2}\right] {\rm sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2$$

$$\times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\,\mu \cos[(l_s + l_i)\beta + \theta]\}$$

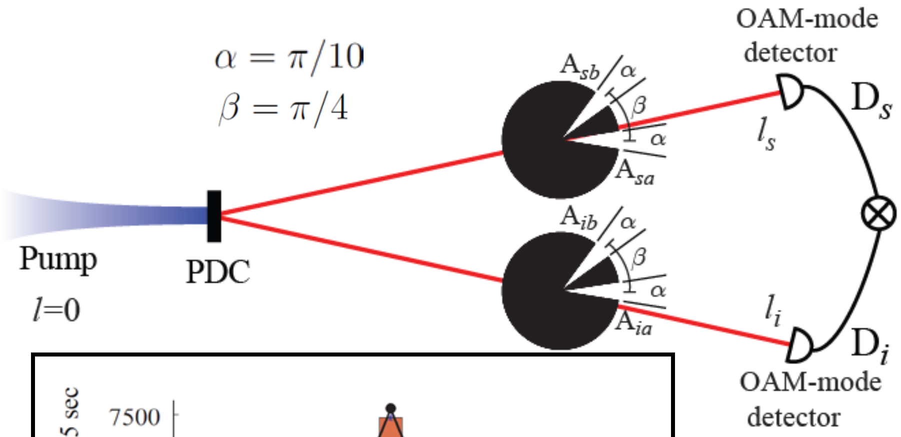
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\,\mu$

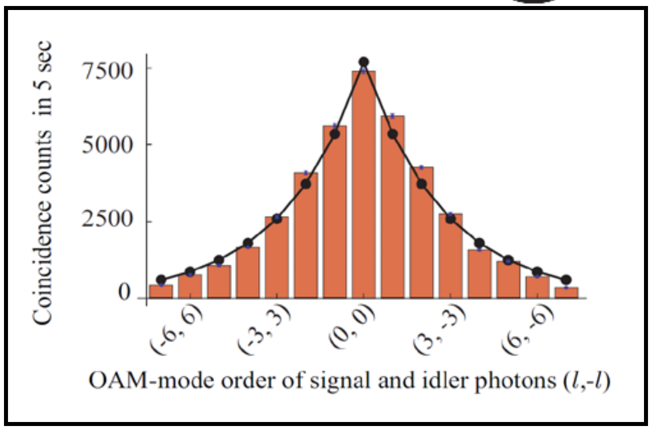
Concurrence of the two-qubit state:

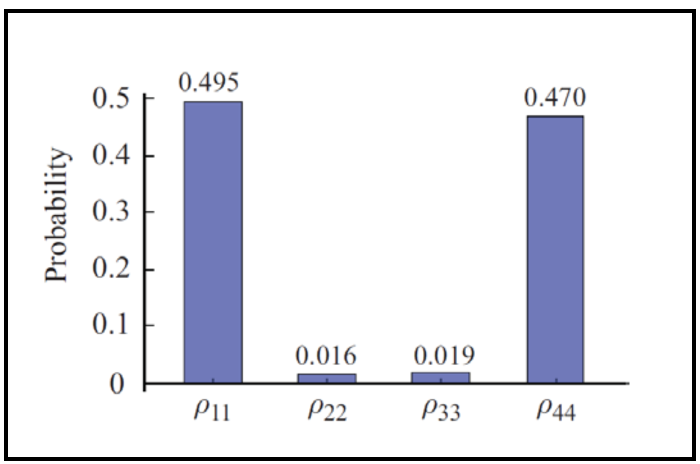
$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\,\mu = V$$

A. K. Jha et al., PRL **104,** 010501 (2010)

Angular Two-Photon Interference







State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{14} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho_{41}^* = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

$$\rho_{11} + \rho_{44} = 1$$

Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2 \times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\ \mu \cos[(l_s + l_i)\beta + \theta]\}$$

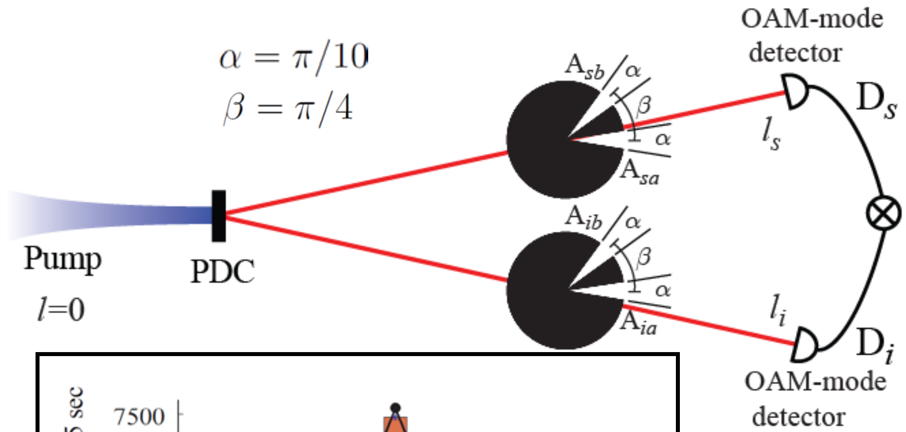
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

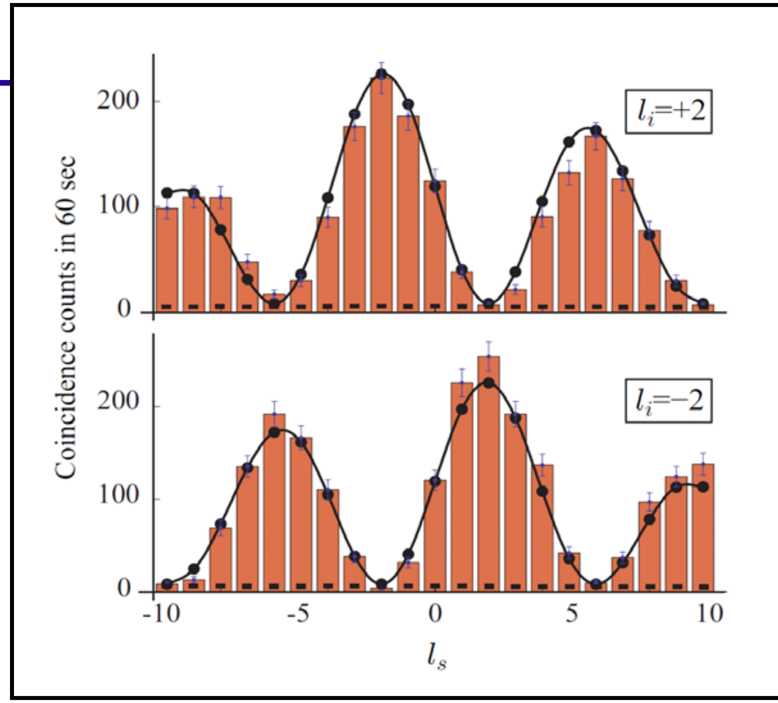
Concurrence of the two-qubit state:

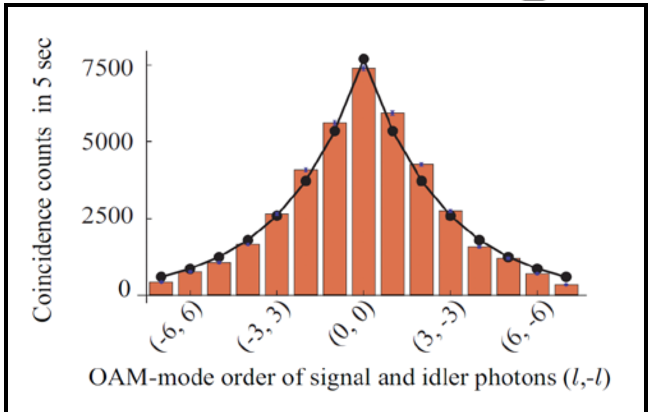
$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\ \mu = V$$

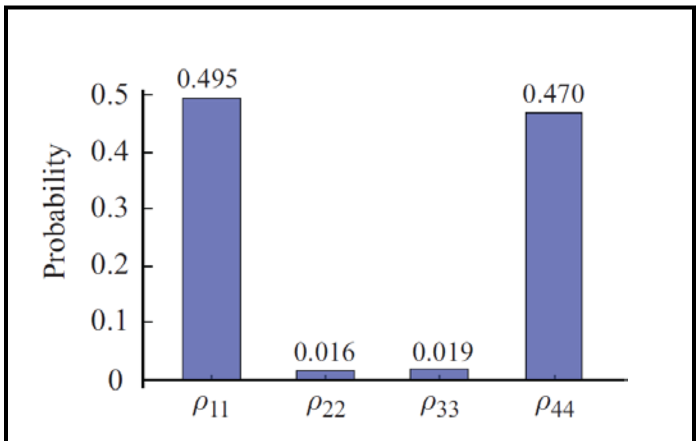
A. K. Jha et al., PRL **104,** 010501 (2010)

Angular Two-Photon Interference









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$$\times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\,\mu \cos[(l_s + l_i)\beta + \theta]\}$$

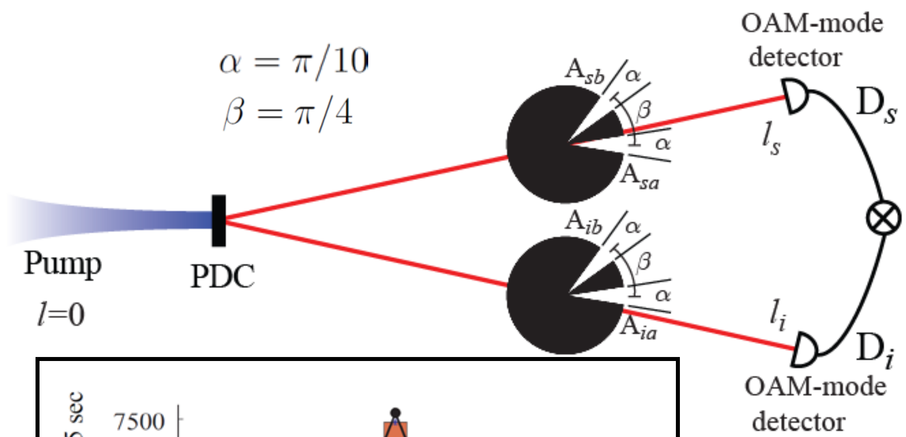
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\,\mu$

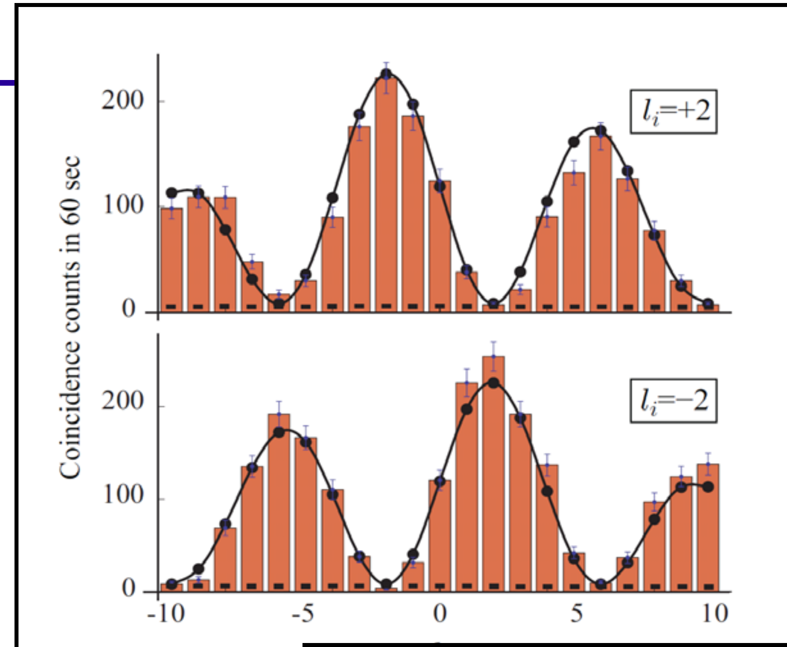
Concurrence of the two-qubit state:

$$C(\rho_{\mathrm{qubit}}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\,\mu = V$$

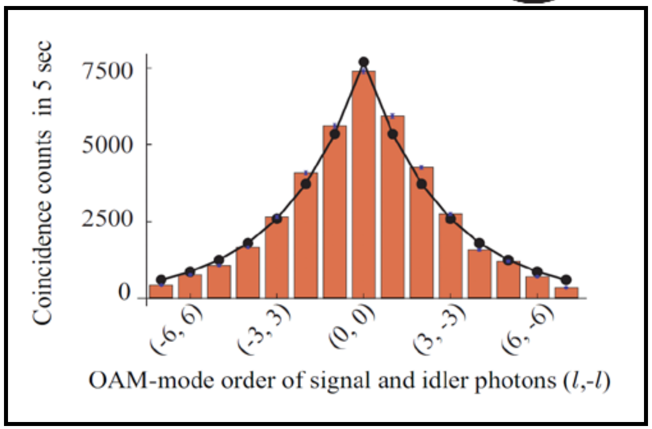
A. K. Jha et al., PRL **104,** 010501 (2010)

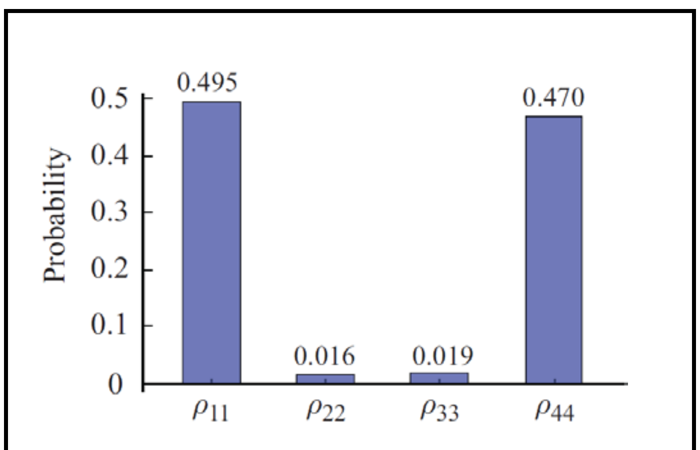
Angular Two-Photon Interference





$$C(\rho_{\text{qubit}}) = 0.963$$





Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,\text{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \text{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2$$

$$\times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\;\mu \cos[(l_s + l_i)\beta + \theta]\}$$

Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\;\mu$

Concurrence of the two-qubit state:

$$C(\rho_{\text{qubit}}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\;\mu \; = V$$

A. K. Jha et al., PRL **104,** 010501 (2010)
Jha, Agarwal, and Boyd, PRA83, 053829 (2011)

Summary and Conclusions

- A unified description of two-photon interference effects in terms of two-photon path length difference (ΔL) and two-photon path-asymmetry length difference ($\Delta L'$).
- HOM effect was described as the change in two-photon coherence as a function of two-photon path-asymmetry length difference ($\Delta L'$).
- Studied angular two-photon interference effects
- Demonstrated an angular two-qubit state

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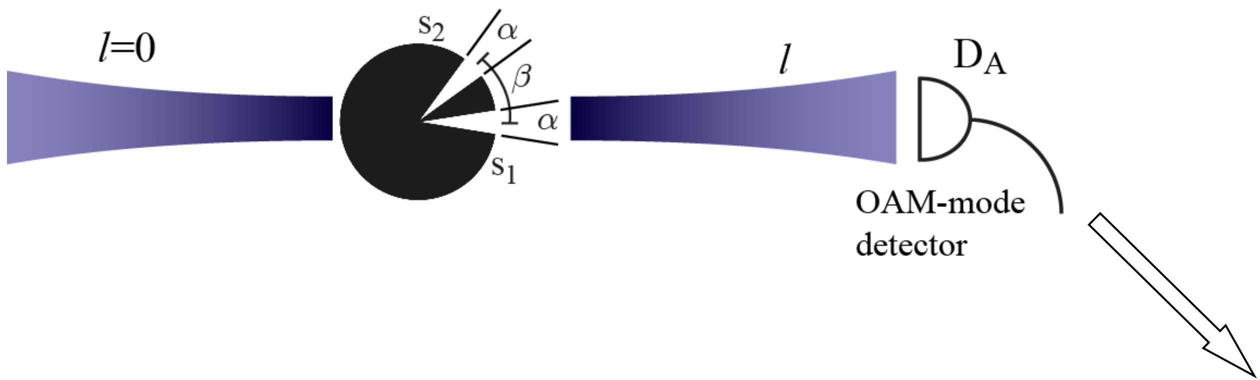
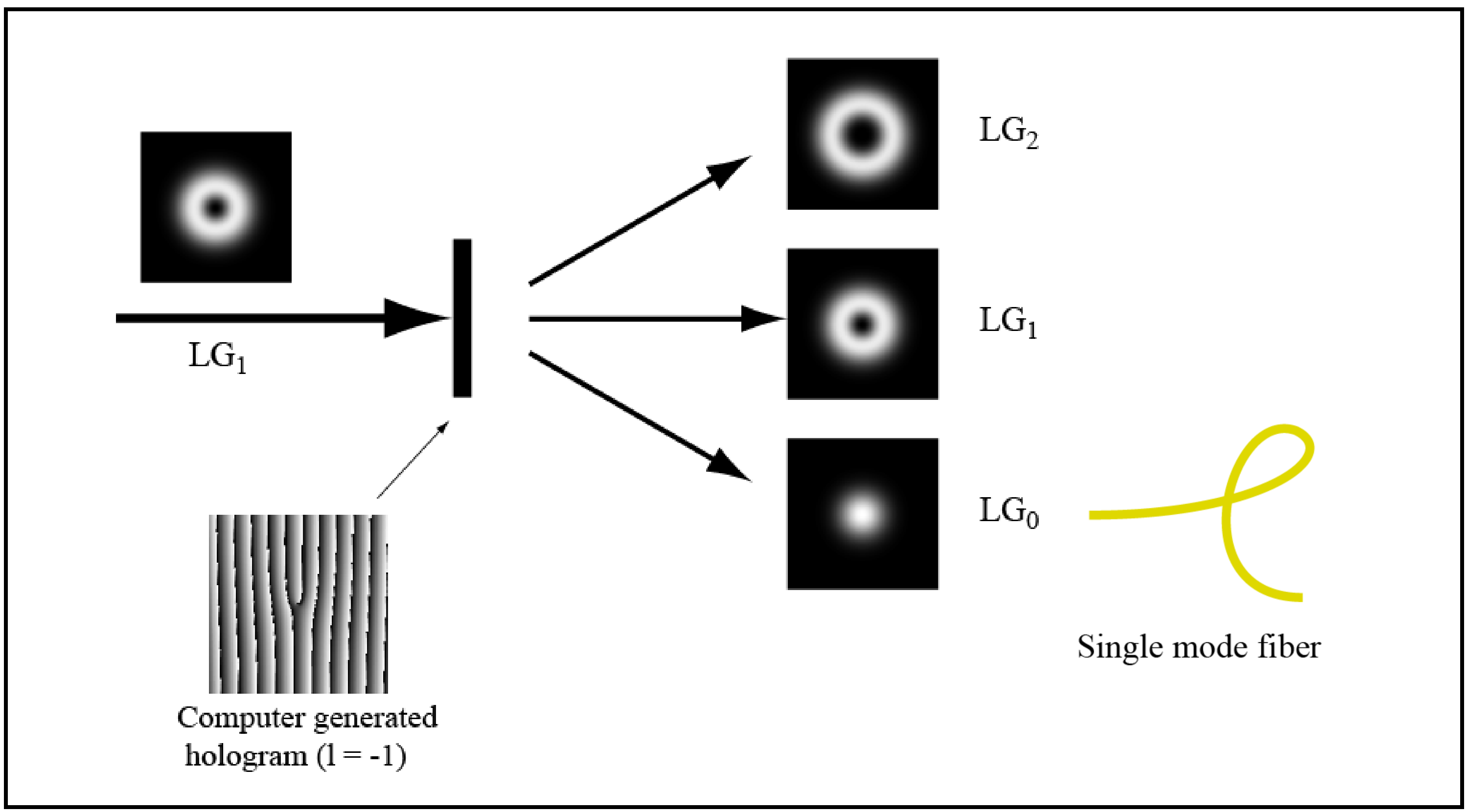
Angular One-Photon Interference





Angular One-Photon Interference